Diversity-Multiplexing Tradeoff in Cooperative Wireless Systems

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***Abstract*— We first examine a system with a single source-destination pair and two relays, each node with a single antenna, and explore whether this virtual multi-input multi-output (MIMO) system can mimic a physical MIMO in terms of diversity-multiplexing tradeoff (DMT). We show that even under the idealistic assumption of full-duplex relays and a clustered network, the relay system can never fully mimic a real MIMO DMT, it is multiplexing gain limited. The limitation comes from the fact that source and destination are connected to relays with finite capacity links. We provide communication strategies that achieve the best DMT of this relay system. We extend our work to cover cooperative systems with multiple sources and multiple destinations and show that the same limitation is still in effect. Our results suggest that while cooperative relaying is able to provide high spatial diversity for low multiplexing gains, it can never mimic a physical MIMO for large multiplexing gains.**

I. Introduction

Wireless networks are subject to fading. Cooperation among different users provides a flexible alternative to using multiple antennas, creating diversity and mitigating fading. User cooperation can increase achievable rates and decrease susceptibility to channel variations [9], [10].

In fading channels an important performance measure of communication schemes is the diversity gain. Diversity gain describes how fast the probability of error decays with increasing signal to noise ratio (SNR). If the system can utilize independently faded channels to send replicas of the same signal, it becomes more reliable. In [9], [10] it was shown that via user cooperation higher diversity gains can be achieved. Laneman *et.al.* proposed several cooperative schemes for a half-duplex single relay system that result in higher diversity gains [7]. In [14], we investigated a two relay system from a diversity point of view.

Wireless communication systems and strategies are also characterized by their multiplexing gains. The multiplexing gain shows how fast the actual rate of the system increases with increasing SNR. The multiplexing gain is limited by the degrees of freedom available in the system. For example a multiple-input multiple-output (MIMO) system can be considered as a collection of parallel spatial channels. One could use the parallel channels to repeat the same signal for higher reliability or to send independent information streams for a higher rate. The fundamental tradeoff between these two types of gains -diversity and multiplexing- is established in [16] for MIMO and is a powerful performance criterion for comparing different MIMO strategies.

User cooperation is usually considered as a viable substitute for multiple antennas. Therefore, its diversity-multiplexing tradeoff (DMT) becomes an important problem. In [1], the authors investigate the DMT of some half-duplex cooperative schemes with single antenna nodes when all inter-user channels are fading. Current literature on cooperation, [1], [7]-[11], [14] to list a few, illustrates that cooperative relaying can behave like a transmit or a receive antenna array. However, if the multiple relays could be configured so that the overall system acts like MIMO, both the diversity and the multiplexing gains would be higher.

In [15], we showed that in order to have maximal MIMO diversity gain for a single source-destination pair and multiple relays, the relays should be clustered around the source and the destination evenly. In other words, half of the relays should be in close proximity to the source and the rest close to the destination so that they have a strong inter-user channel approximated as additive white Gaussian noise (AWGN). Only for this clustered case we can get maximal MIMO diversity, any other placement of relays results in lower diversity gains. However, [15] investigates the problem only from a diversity perspective. On the other hand, in [5], the authors only examine the multiplexing gain of a cooperative system. In this work, we consider the complete DMT of relay and cooperative systems. We show that there are fundamental limitations with respect to MIMO systems. We find that even with full-duplex relays all relay systems fall short of MIMO. The same problem persists in cooperative systems with multiple source-destination pairs as well.

In Section II, we introduce the channel model. In Section III, we provide upper bounds for the DMT for relay systems. We present the schemes that achieve these upper bounds in Section IV. Section V extend the work to cover a fully cooperative system. Finally, Section VI provides conclusions.

II. System Model

In this section we examine a system with two relay nodes and a single source-destination pair. We consider a two-source two-destination problem in Section V. Usually all inter-node

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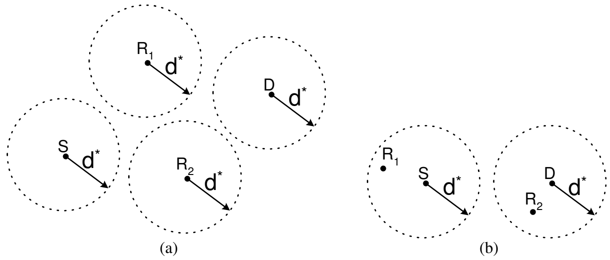

Fig. 1. (a) Non-clustered: All nodes are in Rayleigh zones, (b) Clustered: R_1 and R_2 are in AWGN zones of the source and the destination respectively.

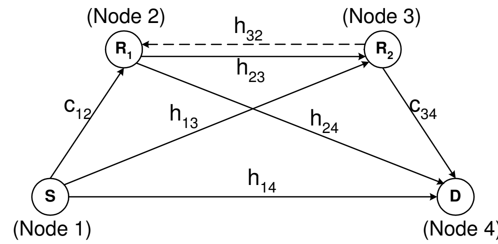

Fig. 2. A two-relay system

channels are modelled as having Rayleigh fading. We assume fading is slow and frequency non-selective. However, when two nodes are very close to each other, Rayleigh channel model assumption is not valid anymore as the line of sight component in the received signal gets stronger. We will model this as an AWGN channel. The threshold distance for which AWGN model becomes accurate is denoted as d^*. There is a dead zone around the nodes so that the AWGN channel gain is bounded. Motivated by [15] we will consider two scenarios. Non-clustered, i.e. all nodes are in Rayleigh zones (Fig. 1(a)) and clustered, i.e. one relay is clustered with the source and the other is clustered with the destination (Fig. 1(b)).

The system model and the channel gains associated with each link for the two relay system are given in Fig. 2. The received signals are

$$\begin{aligned} Y_2 &= c_{12}X_1 + h_{32}X_3 + Z_2 \\ Y_3 &= h_{13}X_1 + h_{23}X_2 + Z_3 \\ Y_4 &= h_{14}X_1 + h_{24}X_2 + c_{34}X_3 + Z_4 \end{aligned}$$

where X_i and Y_i are transmitted and received signals at node i respectively. The channel gains $h_{ij},\ i,j = 1,2,3,4$ are independent, zero mean complex Gaussian with real and imaginary parts each with variance σ_{ij}^2, which is proportional to $1/\sqrt{d_{ij}^{\alpha}}$, where d_{ij} is the distance between nodes i and j and α is the path loss exponent. If the system is not clustered, then the channel gains c_{ij} are also Rayleigh. On the other hand, if the system is clustered, then c_{12} and c_{34} are equal to $\sqrt{G_{12}}$ and $\sqrt{G_{34}}$ respectively, which are the AWGN channel gains. The source, the first relay, R_1, and the second relay, R_2, have power constraints P_1, P_2 and P_3 respectively. These power levels are adjusted to have similar received SNR at the destination. The noise terms at the receivers are iid complex Gaussian with zero mean and variance $\mathcal{N}_o$.

We would like to find the most optimistic scenario in terms of DMT for the relay system, hence we assume the relays in our system are full-duplex, that is they can transmit and receive at the same time in the same band. Because of this assumption, for a non-clustered system R_1 can overhear R_2's signal while listening to the source. However, when the relays are clustered, R_2's signal at R_1 is very weak. As the nodes aim comparable received SNR values at the destination, at R_1 the source signal is received much stronger than the R_2 signal. Therefore, in Fig. 2 the node 3 to node 2 link is shown dashed, indicating it is present only if nodes are not clustered. Note that this is the level-set approach of [4].

In our system all receivers have channel state information (CSI) about their incoming channels. In addition for the clustered system R_2 and the destination know each other's incoming fading levels. This is a reasonable assumption since the clustered nodes can easily exchange channel state information. Also, this assumption makes the system more similar to a real MIMO. Transmitters do not have instantaneous CSI.

The diversity order, d, and the multiplexing gain, r, are defined as

$$\lim_{\mathrm{SNR}\to\infty} \frac{\log P_e(\mathrm{SNR})}{\log \mathrm{SNR}} = -d \quad \text{and} \quad \lim_{\mathrm{SNR}\to\infty} \frac{R(\mathrm{SNR})}{\log \mathrm{SNR}} = r$$

where $P_e(\mathrm{SNR})$ is the probability of error and $R(\mathrm{SNR})$ is the actual rate of the system, both as functions of SNR [16]. When the rate of the system $R(\mathrm{SNR})$ increases like $r \log \mathrm{SNR}$ we need to find the decay rate of the probability of error to find the diversity order d corresponding to the multiplexing gain r.

III. Upper Bounds

The maximum rate of information flow from the source to the destination is limited by the minimum cut [2, Theorem 14.10.1]. The cut-sets of interest are shown in Fig. 3. In this section we use T_1, T_2 and T_4 to obtain tight bounds. In the next section we will show that this bound is achievable for both non-clustered and clustered systems.

From the cut-sets T_1, T_2 and T_4 we can upper bound any achievable rate R_a of the system for given fading levels as

$$\begin{aligned} R_a &\leq I_{T_1} = I(X_1; Y_2Y_3Y_4|X_2X_3) && (1) \\ R_a &\leq I_{T_2} = I(X_1X_2; Y_3Y_4|X_3) && (2) \\ R_a &\leq I_{T_4} = I(X_1X_2X_3; Y_4). && (3) \end{aligned}$$

In a fading channel the dominant error event is the outage event. When there is outage, it is very probable that a frame error occurs and when there is no outage, we can make the probability of frame error sufficiently small by adjusting the channel coding scheme in use. In [16] it is explicitly proven that the probability of error is on the same order as the probability of outage at high SNR and we can deduce the DMT of the system via the outage probability. Therefore, instead of finding the probability of error behavior of a system, it is sufficient to study the outage behavior only.

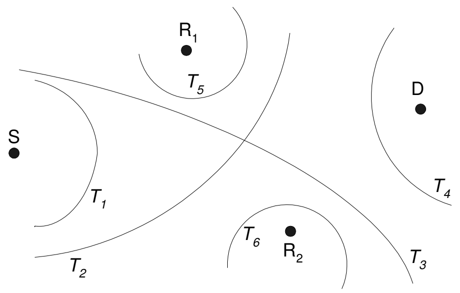

Fig. 3. Cut-sets. Note that T_5 and T_6 are necessary only for the two-source two-destination problem of Section V.

For a given channel realization and for any coding scheme achieving R_a we can write for cut-set T_i

$$\begin{aligned} P(R_a < R) &\geq P(I_{T_i} < R) \\ &\geq \min_{p(x_1,x_2,x_3)} P(I_{T_i} < R) \\ &= P_{out,T_i}. \end{aligned}$$

Hence, we have $P(R_a < R) \geq \max_i P_{out,T_i}$. Since this is true for any coding scheme achieving R_a, we can minimize the left hand side over all coding schemes and lower bound the probability of outage, P_{out}, as

$$P_{out} = \min_{\text{all coding schemes } \mathcal{C}} P(R_a < R) \geq \max_i P_{out,T_i}.$$

As a result, the cut-sets present a lower bound on the probability of outage and consequently an upper bound on the diversity order of the actual system, i.e. $d \leq \min\{d_{T_i}\}$ where d_{T_i} is the diversity order of cut-set T_i.

For the non-clustered system

$$I_{T_1} = \log\left(1 + |h_{12}|^2 \frac{P_1}{\mathcal{N}_o} + |h_{13}|^2 \frac{P_1}{\mathcal{N}_o} + |h_{14}|^2 \frac{P_1}{\mathcal{N}_o}\right) \quad (4)$$

$$I_{T_2} = \log\det\left(I_2 + \mathbf{A}Q\mathbf{A}^\dagger\right) \quad (5)$$

$$I_{T_4} = \log\left(1 + |h_{14}|^2 \frac{P_1}{\mathcal{N}_o} + |h_{24}|^2 \frac{P_2}{\mathcal{N}_o} + |h_{34}|^2 \frac{P_3}{\mathcal{N}_o}\right) \quad (6)$$

where $\mathbf{A} = \begin{pmatrix} h_{13} & h_{23} \\ h_{14} & h_{24} \end{pmatrix}$ and $Q = \begin{pmatrix} \frac{P_1}{\mathcal{N}_o} & 0 \\ 0 & \frac{P_2}{\mathcal{N}_o} \end{pmatrix}$. We can easily see that in this case cut-sets T_1 and T_4 become limiting. Equation (5) mimics a 2 by 2 MIMO system performance, whereas (4) and (6) mimic a 1 by 3 (or 3 by 1) system. Since for each multiplexing gain a 1 by 3 system has lower diversity than a 2 by 2, the DMT of the non-clustered system is $3(1-r)$, which is shown in Fig. 4.

For the clustered system, these cut-sets result in

$$I_{T_1} = \log\left(1 + G_{12} \frac{P_1}{\mathcal{N}_o} + |h_{13}|^2 \frac{P_1}{\mathcal{N}_o} + |h_{14}|^2 \frac{P_1}{\mathcal{N}_o}\right) \quad (7)$$

$$I_{T_2} = \log\det\left(I_2 + \mathbf{A}Q\mathbf{A}^\dagger\right) \quad (8)$$

$$I_{T_4} = \log\left(1 + |h_{14}|^2 \frac{P_1}{\mathcal{N}_o} + |h_{24}|^2 \frac{P_2}{\mathcal{N}_o} + G_{34} \frac{P_3}{\mathcal{N}_o}\right). \quad (9)$$

In contrast to the non-clustered system, now cut-sets T_1 and T_4 do not limit the diversity for multiplexing gains up to 1. Note that (7) is always larger than $\log(1 + G_{12}P_1/\mathcal{N}_o)$, which is the capacity of an AWGN channel with gain $\sqrt{G_{12}}$. This means it is possible to operate at this positive rate reliably without any outage. And as P_1 increases, this operational rate increases as $\log(P_1/\mathcal{N}_o)$. Therefore, we conclude that the first cut-set results in infinite diversity for all multiplexing gains up to 1. A similar argument holds for (9) since it is always larger than $\log(1 + G_{34}P_3/\mathcal{N}_o)$.

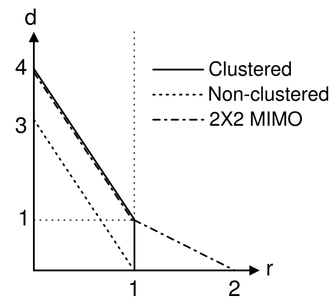

Fig. 4. Diversity-multiplexing tradeoff of a two-relay system

On the other hand, cut-set T_2 makes the system equivalent to a 2 transmit antenna 2 receive antenna system and this limits the diversity gains. The DMT of this term is $d^*_{22}(r)$ [16] where

$$d_{22}(k) = (2-k)(2-k), \quad k = 0, 1, 2$$

and $d^*_{22}(r)$ is the collection of the lines connecting the points $d_{22}(k)$. Overall for all multiplexing gains less than 1, and the system is limited by the 2 by 2 MIMO performance. The system does not support multiplexing gains larger than 1.

The clustered DMT upper bound is shown in Fig. 4. The multiplexing gain limitation is mainly due to the fact that there is only one source and/or one destination each with a single antenna. When we operate a relay system as a transmit antenna array or as a receive antenna array only, we do not realize this multiplexing gain limitation. However, trying to imitate a real MIMO, we observe that the relay system does not fully offer a *virtual* MIMO. Note that same multiplexing limitations occur when the source has two antennas and a single antenna relay is clustered with the single antenna destination or the symmetric case when the destination has two antennas and a single antenna relay is clustered with the single antenna source.

In the next section we will show that this bound is indeed achievable.

IV. Achievability

A. Clustered System

To prove achievability of the DMT, we use the transmission scheme presented in [3]. For completeness, we briefly describe the transmission and decoding strategies. For R_1 we use decode-and-forward since source to R_1 channel does not present a bottleneck with respect to the diversity-multiplexing upper bound derived in Section III. For R_2, we need to pass along soft information to the destination in order not to lose diversity or multiplexing gains so we use compress-and-forward. To achieve this in a full-duplex system, the source node and the first relay together perform block Markov superposition encoding. The second relay does Wyner-Ziv type compression with side information taken as the destination's received signal, as we have observed that a simple compression scheme without utilizing the side information does not work for DMT. We have B blocks each of which are length n. We assume both B and n are large and the fading remains constant for all B blocks. To decode backwards decoding is used. Under this scheme the following rate R is achievable [3]

$$R < \min\left\{I(X_1;Y_2|X_2), I(X_1X_2;\hat{Y}_3Y_4|X_3)\right\} \tag{10}$$

subject to

$$I(\hat{Y}_3;Y_3|X_3Y_4) \leq I(X_3;Y_4) \tag{11}$$

where the joint probability distribution is $p(x_1,x_2)p(x_3)$ $p(\hat{y}_3|x_3,y_3)p(y_2,y_3,y_4|x_1,x_2,x_3)$. The details of the proof can be found in [3], [6].

Our goal is to argue that the above achievable rate meets the diversity-multiplexing upper bound of Section III. Choosing X_1, X_2 and X_3 independent with complex Gaussian with variances P_1, P_2 and P_3 respectively and $\hat{Y}_3 = Y_3 + \hat{Z}_3$, where $\hat{Z}_3$ is an independent complex Gaussian random variable with zero mean, variance $\hat{N}_3$ and independent from all other random variables, the mutual information terms in equation (10) become

$$I(X_1;Y_2|X_2) = \log\left(1+G_{12}\frac{P_1}{\mathcal{N}_o}\right) \tag{12}$$

$$I(X_1X_2;\hat{Y}_3Y_4|X_3) = \tag{13}$$

$$= \log\left(\frac{\det\left(I_2+\mathbf{A}Q\mathbf{A}^\dagger\right)+\hat{N}_3\left(1+|h_{14}|^2\frac{P_1}{\mathcal{N}_o}+|h_{24}|^2\frac{P_2}{\mathcal{N}_o}\right)}{1+\hat{N}_3}\right).$$

The mutual information in the compression rate constraint of equation (11) are

$$I(\hat{Y}_3;Y_3|X_3Y_4) =$$

$$= \log\left(1+\frac{\det\left(I_2+\mathbf{A}Q\mathbf{A}^\dagger\right)}{\left(1+|h_{14}|^2\frac{P_1}{\mathcal{N}_o}+|h_{24}|^2\frac{P_2}{\mathcal{N}_o}\right)\hat{N}_3}\right) \tag{14}$$

$$I(X_3;Y_4) = \log\left(1+\frac{G_{34}\frac{P_3}{\mathcal{N}_o}}{1+|h_{14}|^2\frac{P_1}{\mathcal{N}_o}+|h_{24}|^2\frac{P_2}{\mathcal{N}_o}}\right).$$

Then the compression noise power has to satisfy

$$\hat{N}_3 \geq \frac{\det\left(I_2+\mathbf{A}Q\mathbf{A}^\dagger\right)}{G_{34}\frac{P_3}{\mathcal{N}_o}}. \tag{15}$$

We choose the smallest $\hat{N}_3$ that satisfies the constraint to obtain the largest achievable rate in equation (10). Note that this compression noise variance, $\hat{N}_3$, depends on the fading levels $|h_{14}|^2$ and $|h_{24}|^2$. That's why R_2 needs the channel channel state information of the destination. In addition, as $\hat{N}_3$ depends on $|h_{13}^2|$ and $|h_{23}^2|$, the destination needs the channel state information available at R_2 to decompress X_3 reliably. Then, if the source to R_1 channel is not limiting and $\hat{N}_3$ satisfies equation (15), the mutual information in equation (13) is achievable. Next we will prove that this mutual information displays a 2 by 2 system behavior for all multiplexing gains smaller than 1.

To prove the DMT we need to find how probability of error decays with increasing SNR when the rate increases as $R = r\log\text{SNR}$, where without loss of generality SNR is defined as the common average received signal to noise ratio at the destination, i.e. $\text{SNR} = d_{14}^{-\alpha}P_1/\mathcal{N}_o = d_{24}^{-\alpha}P_2/\mathcal{N}_o = G_{34}P_3/\mathcal{N}_o$.

Let $\mathcal{E} = \{I(X_1;Y_2|X_2) < I(X_1X_2;\hat{Y}_3Y_4|X_3)\}$ be the event that the first term in equation (10) is less than the second and $\mathcal{E}^c$ is its complement. Then the probability of error is upper bounded as

$$\begin{aligned} P_e &= P(\text{error}|\mathcal{E})P(\mathcal{E}) + P(\text{error}|\mathcal{E}^c)P(\mathcal{E}^c) \\ &\leq P(\text{error}|\mathcal{E}) + P(\text{error}|\mathcal{E}^c). \end{aligned}$$

As the error events are dominated by outage events, we can further write

$$P_e \dot{\leq} P(\text{outage}|\mathcal{E}) + P(\text{outage}|\mathcal{E}^c). \tag{16}$$

When $\mathcal{E}$ happens the system has infinite levels of diversity for all multiplexing gains up to 1. When $\mathcal{E}^c$ is the case, we need to show that $P(\text{outage}|\mathcal{E}^c)$ decays at least as fast as $d_{22}^*(r)$ with increasing SNR.

Let $K = \det\left(I_2+\mathbf{A}Q\mathbf{A}^\dagger\right)$. After we substitute $\hat{N}_3$ in equation (15) into equation (13) we have

$$\begin{aligned} &I(X_1X_2;\hat{Y}_3Y_4|X_3) = \\ &= \log\frac{K\left(1+|h_{14}|^2\frac{P_1}{\mathcal{N}_o}+|h_{24}|^2\frac{P_2}{\mathcal{N}_o}+G_{34}\frac{P_3}{\mathcal{N}_o}\right)}{K+G_{34}\frac{P_3}{\mathcal{N}_o}} \\ &\geq \log\frac{KG_{34}\frac{P_3}{\mathcal{N}_o}}{K+G_{34}\frac{P_3}{\mathcal{N}_o}} \end{aligned}$$

Then for $R = r\log\text{SNR}$,

$$\begin{aligned} P(\text{outage}|\mathcal{E}^c) &= P(I(X_1X_2;\hat{Y}_3Y_4|X_3) < R) \\ &\leq P\left(\log\frac{KG_{34}\frac{P_3}{\mathcal{N}_o}}{K+G_{34}\frac{P_3}{\mathcal{N}_o}} < R\right). \end{aligned}$$

Substituting the equality $\text{SNR} = G_{34}P_3/\mathcal{N}_o$ and for $\text{SNR} > \beta$ and $\beta > 1$, where β is a constant and $r < 1$, we can further upper bound $P(\text{outage}|\mathcal{E}^c)$ as

$$P(\text{outage}|\mathcal{E}^c) \leq P\left(K < \frac{1}{1-\beta^{r-1}}\text{SNR}^r\right) \doteq \text{SNR}^{-d_{22}^*(r)}.$$

Overall, the second term in equation (16) dominates and hence

$$P_e \dot{\leq} \frac{1}{\text{SNR}^{d_{22}^*(r)}}$$

for all $r < 1$. On the other hand we know that P_e cannot decay faster than $d^*_{22}(r)$ due to the cut-set upper bounds. Thus, we conclude that this relaying scheme achieves the DMT.

In [6, Theorem 4], the authors prove an achievable rate for a multiple relay system, in which some of the relays decode-and-forward and the rest compress-and-forward. Furthermore, the relays that compress-and-forward partially decode the signals from the decode-and-forwarding relays. Performing this partial decoding leads to higher achievable rates. However, to achieve the DMT upper bound, there is no need for partial decoding and a simpler strategy is enough.

For the receive cluster, compress-and-forward fits very well. If the side information at the destination has high power, i.e. its own received signal Y_4 has high quality due to large h_{14} and h_{24}, then R_2 to destination channel has lower capacity because in the decoding process, the side information behaves like interference. On the other hand, the correlation between the relay and destination signals is higher and a coarse description of R_2 signal is enough to help the destination. However, if the side information has low received power, the R_2 to destination channel has higher capacity and as the correlation is less, R_2 can send finer information. Note that decode-and-forward for R_2 does not work. For rates with multiplexing gain 1, if R_2 tries to decode, it has fixed probability of error as it acts like the receiver in a 2 by 1 system. With a fixed probability of error, it does not increase the diversity order at the destination.

B. Non-clustered system

For the non-clustered system even a simpler scheme works: Both relays only listen to the source and decode-and-forward. We assume the source, R_1 and R_2 perform block Markov superposition coding. The signaling structure is shown in Fig. 5. Both relays listen to the source in the first block. If the relays are not in outage, their estimates w'_1 and w''_1 are equal to w_1 with high probability. If a relay is in outage, which happens with probability $1/\text{SNR}^{1-r}$ for multiplexing gain r, it will be in outage for all blocks since channel is slowly fading. In that case that relay stops transmitting. This information is only needed at the destination but not at the source and it can be communicated to the destination at a negligible cost at the beginning of B blocks. In the second block,the source sends $x_1(w_1, w_2)$ and R_1 and R_2 send their own channel codewords $x_2(w'_1)$ and $x_3(w''_1)$. With backward decoding, the destination can jointly decode the source message by processing the source and the active relays. Then the probability of outage

Block 1	Block 2	Block 3		Block B
$x_1(1, w_1)$	$x_1(w_1, w_2)$	$x_1(w_2, w_3)$		$x_1(w_{B-1}, 1)$
$x_2(1)$	$x_2(w'_1)$	$x_2(w'_2)$	...	$x_2(w'_{B-1})$
$x_3(1)$	$x_3(w''_1)$	$x_3(w''_2)$		$x_3(w''_{B-1})$

Fig. 5. A relaying scheme that achieves the diversity-multiplexing tradeoff upper bound for the non-clustered system. Both relays do decode-and-forward. The figure shows the case when both relays can decode.

for this system, when the multiplexing gain is r, is equal to

$$\begin{aligned} P_{out} &= P(\text{outage}|\text{both relays decode})P(\text{both relays decode}) \\ &\quad + P(\text{outage}|\text{one of the relays decode}). \\ &\qquad\qquad P(\text{one of the relays decode}) \\ &\quad + P(\text{outage}|\text{none of the relays decode}). \\ &\qquad\qquad P(\text{none of the relays decode}). \end{aligned}$$

This outage probability becomes

$$\begin{aligned} P_{out} &\doteq \text{SNR}^{-3(1-r)}1 + \text{SNR}^{-2(1-r)}\text{SNR}^{-(1-r)} \\ &\qquad + \text{SNR}^{-(1-r)}\text{SNR}^{-2(1-r)} \\ &\doteq \text{SNR}^{-3(1-r)} \end{aligned}$$

at high SNR, which is the outage behavior of a 1 by 3 system (or 3 by 1) system.

The achievable rate presented in [6, Theorem 1] is higher as R_2 listens to both the source and R_1 signals. However, with this proposed signaling structure, relays ignore each other's signals. This is enough to meet the diversity-multiplexing upper bound.

V. Two-Source Two-Destination Cooperative System

In Section III, we observed that neither a two-relay system with one source-destination pair (each equipped with single antennas) nor the single antenna source, single antenna relay and two-antenna destination system work like MIMO. They are both multiplexing gain limited. However, if in the latter case the relay were a source itself, and the sources cooperated, the system would achieve the complete diversity-multiplexing gain of MIMO [12]. One wonders what would happen if we replaced the two-antenna destination with a single antenna relay and a single antenna destination with a finite capacity link in between. Therefore, we examine the two-source two-destination problem, in which sources cooperate in transmission and destinations cooperate in reception. We will call nodes 1, 2, 3 and 4 as S_1, S_2, D_2 and D_1 respectively. The system model is shown in Fig. 6.

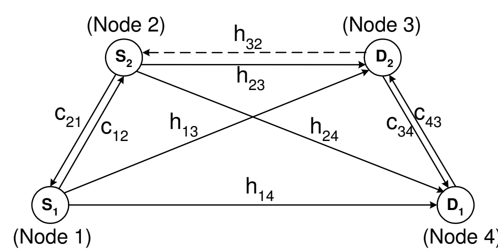

Fig. 6. A two-source two-destination system

First we examine the case when both destinations decode both sources. This is analogous to MIMO systems and represents the information transfer from a group of antennas to another group of antennas. We require both destinations have the same diversity level. Using the cut-set bounds of Fig. 3 we have

$$R_1 \leq I_{T_1} = I(X_1; Y_2Y_3Y_4|X_2X_3X_4) \quad (17)$$
$$R_2 \leq I_{T_5} = I(X_2; Y_1Y_3Y_4|X_1X_3X_4) \quad (18)$$
$$R_1 + R_2 \leq I_{T_2} = I(X_1X_2; Y_3Y_4|X_3X_4) \quad (19)$$
$$R_1 + R_2 \leq I_{T_4} = I(X_1X_2X_3; Y_4|X_4) \quad (20)$$
$$R_1 + R_2 \leq I_{T_6} = I(X_1X_2X_4; Y_3|X_3) \quad (21)$$

where R_1 and R_2 are the information rates of S_1 and S_2 respectively. Cut-sets T_4 and T_6 limit the sum rate multiplexing gain received by each destination by 1 as mutual information terms in equations (20) and (21) are multiplexing gain limited for both clustered and non-clustered systems. For multiplexing gains up to 1, 2 by 2 MIMO behavior is the upper bound to the DMT for a clustered system (equation (19)), 1 by 3 SIMO or 3 by 1 MISO behavior constitutes the upper bound for a non-clustered system, as cut-sets T_4 and T_6 again limit the sum multiplexing gain. As a result, even if there are two sources and two destinations in the system, we cannot increase the sum multiplexing gain.

To achieve these upper bounds in the full-duplex system the sources mutually need to act as relays for each other. In addition, the destinations need to help each other mutually. For this purpose, the sources employ the coding scheme of Willems [13] which is also applied in [9] to achieve transmitter cooperation. This coding technique allows the two sources to decode-and-forward for each other simultaneously. The destinations compress-and-forward for each other. Decoding is backwards. A detailed illustration of the signal structure and achievable rates are omitted due to space limitations.

For the non-clustered cooperative system the destination nodes need not compress-and-forward. The simpler decode-and-forward strategy works as well. This will not result a degradation in the achievable DMT as in Section IV-B.

Recall that the above analysis requires both destinations to decode both sources. However, this condition might be too strict and limit the multiplexing gain of the system. In the cooperative interference channel, D_1 is only required to decode S_1 and D_2 to decode S_2. The cooperative interference channel imposes looser decoding requirements on the destinations and potentially leads to higher achievable rates. However, in [5], the authors prove that the multiplexing gain of this system is also limited by 1. Therefore, the diversity-multiplexing gain upper bound is the same as the case when both destinations decode both sources. Note that the same coding strategy as above still achieves the diversity-multiplexing upper bound as the strict assumption of decoding both sources can only make the achievable region smaller.

VI. Conclusions and Future Work

In this work we compare wireless relay and cooperative networks with a physical MIMO system. We show that despite the common belief that the relay or cooperative systems can behave like a *virtual* MIMO, this is not possible for all multiplexing gains. Both for relay and cooperative systems, even if the nodes are clustered, the finite capacity link between nodes in the source cluster and the finite capacity link between the nodes in the destination cluster are bottlenecks and limit the achievable multiplexing gain of the system. Cooperative interference channels are also limited in the same way. It is straightforward to extend our results for a single source-destination pair with multiple relays and for cooperative systems with N sources and N destinations with each destination decoding all sources. However, it is challenging to extend the system to a general network with N nodes in the source cluster (some of which are sources and the rest relays) and M nodes in the receiver cluster (some are destinations and some are mere relays). Future work also includes extending the work to systems with multiple antenna elements and to half-duplex systems.

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