Diversity Gains and Clustering in Wireless Relaying

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Abstract — **We consider a wireless system consisting of one source, one destination and M relays. Assuming path loss and Rayleigh fading, we use the cutset upper bound to show that no matter where the relays are located, the maximum diversity one can obtain is $M+1$. However, one can achieve a higher diversity gain, namely $\lfloor\left(\frac{M+2}{2}\right)^2\rfloor$, if $\lfloor\frac{M}{2}\rfloor$ of the relays are clustered with the source and $\lceil\frac{M}{2}\rceil$ with the destination. This result utilizes the observation that if two wireless nodes are very close, Rayleigh assumption breaks and the proper channel model is additive white Gaussian noise (AWGN). Hence to realize a *virtual multi-input multi-output* (MIMO) system, clustering is essential.**

I. Diversity upper bound for the Rayleigh zone

Consider a system with one source (S), two relays (R_1 and R_2) and one destination (D). Let the nodes be numbered as 1, 2, 3 and 4 respectively. All channels have path loss and fading as long as the distance between two terminals d is greater than ϵ. When d is less than ϵ, the two terminals have a strong line of sight component and AWGN model with no fading is more appropriate. Hence we use the term *Rayleigh zone* when distance between two nodes is greater than ϵ, *AWGN zone* otherwise. In this section, all nodes are assumed to be in Rayleigh zones and we assume complex Gaussian fading that is independent for different channels.

We consider a channel gain matrix $\mathbf{A}$ where each entry a_{ij} denotes the channel gain between node i and j, $i=1,2,3$ and $j=2,3,4$. For a given $\mathbf{A}$, if a coding scheme $\mathcal{C}$ has an achievable rate $R(\mathbf{A})$ from source to destination using the relays, then it is upper bounded by

$$\begin{aligned} R(\mathbf{A}) \quad \leq \quad & \min\{I(X_1;Y_2,Y_3,Y_4|X_2,X_3,\mathbf{A}), \\ & I(X_1,X_2;Y_3,Y_4|X_3,\mathbf{A}), I(X_1,X_3;Y_2,Y_4|X_2,\mathbf{A}), \\ & I(X_1,X_2,X_3;Y_4|\mathbf{A})\} \qquad (1) \end{aligned}$$

for some $p(x_1,x_2,x_3)$ using Theorem 14.10.1 in [1]. Here X_i and Y_i are the transmitted and received signals by node i, $i=1,2,3,4$. In the above expression the k^{th} mutual information term I_{S_k} corresponds to cutset S_k, where $S_1=\{S\}$, $S_2=\{S,R_1\}$, $S_3=\{S,R_2\}$, and $S_4=\{S,R_1,R_2\}$. Note that we assume no channel state information at the transmitters. Defining P_{out,S_k} as the outage probability for cutset k, we have for all k

$$\begin{aligned} P(R(\mathbf{A})<R) \quad & \geq \quad P(I_{S_k}<R) \\ & \geq \quad \min_{p(x_1,x_2,x_3)} P(I_{S_k}<R) \\ & = \quad P_{out,S_k}. \qquad (2) \end{aligned}$$

Therefore $P(R(\mathbf{A})<R) \geq \max_k P_{out,S_k}$ and

$$P_{out} = \min_{\text{all coding schemes } \mathcal{C}} P(R(\mathbf{A})<R) \geq \max_k P_{out,S_k}. \qquad (3)$$

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Hence, the largest outage term among the cutsets is the tightest lower bound to the outage probability.

For cutset S_1, the system is equivalent to a 1 transmitter, 3 receiver MIMO system and P_{out,S_1} decays as SNR^{-3} for large SNR. The same also holds for cutset S_4, where the system is equivalent to 3 transmitters and 1 receiver. For cutsets S_2 and S_3 the system behaves like a 2x2 MIMO system and hence the diversity level is 4. These observations suggest that the slowest decay rate for outage probability among cutsets is SNR^{-3} and the maximal diversity order is 3.

II. An Example for Clustering

In this section we argue clustering increases the diversity gains and achieves 4 levels of diversity for the two relay system. For simplicity we assume that the relays cannot receive and transmit at the same time and both relays amplify and forward their incoming signals. There is time division between the transmitters. Source transmits in the first time slot. This transmission is received by both relays and the destination. In the second time slot, R_1, which we assume as the relay closer to the source, relays this information. In the last time slot, R_2 combines signals from S and R_1 and forwards it to the destination. Finally the destination combines all the three signals to decode. Consistent with Section I, it was shown in [3] that if both relays are in the Rayleigh zone, then irrespective of their locations, the destination observes 3 levels of diversity. However, using arguments similar to those of [3], we prove that the only way to obtain a diversity order of 4 is to have R_1 in the AWGN zone of the source *and* to have R_2 in the AWGN zone of the destination. Note that this result is different from the single relay case, for which 2 levels of diversity are obtained no matter where the relay is located [2]. Since the diversity level dominates outage performance at high SNR, we conclude that clustering is outage optimal for high SNR values and results in a virtual MIMO system.

The above analysis can be easily extended to a system with M relays to obtain $\lfloor\left(\frac{M+2}{2}\right)^2\rfloor$ levels of diversity. Although our example considers an amplify and forward scheme, similar arguments can be made for other relaying schemes as well. As the cutset bound indicates, the limiting assumption to diversity is the Rayleigh fading rather than the relaying scheme. As long as relay to D or S to relay channels are bottlenecks, the system cannot mimic a MIMO spatial diversity system.

References

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