Diversity-Multiplexing Tradeoff in Multiple-Antenna Relay Systems

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***Abstract*— We study the diversity-multiplexing tradeoff (DMT) for the full-duplex relay channel when the source and the destination have multiple antennas, and the relay has 1 or more. We find DMT upper bounds and investigate the achievable performance of decode-and-forward (DF), partial decode-and-forward (PDF), and compress-and-forward (CF) protocols. We study the effect of increased degrees of freedom in the direct link and the source-relay channel when multiple antennas are introduced. Our results suggest that while DF is DMT optimal when all terminals have one antenna each, it cannot maintain its good performance when the degrees of freedom in the direct link is increased. CF proves to be a more robust strategy, which works well in multi-antenna scenarios studied in this paper. We also extend our results for clustered relay networks to find DMT upper bounds and achievable performances.**

I. Introduction

Fading in the wireless channel decreases communication reliability. Multiple-input multiple-output (MIMO) systems introduce spatial diversity and combat fading. Additionally, taking advantage of the rich scattering environment, MIMO increases spatial multiplexing. The fundamental tradeoff between reliability and spatial multiplexing in MIMO systems, called the diversity-multiplexing tradeoff (DMT), is established in [11] for high signal to noise ratio (SNR).

User cooperation becomes a practical alternative to MIMO when the size of the wireless device is limited. Similar to MIMO, cooperation among different users can increase reliability and achievable rates [7]. In [5], the authors study a half-duplex single relay system and show that higher diversity gains are achievable. User cooperation literature mostly focuses on diversity gains, which shows that multi-input single-output (MISO) or single-input multi-output (SIMO) like diversity gains are achievable for user cooperation.

Some of the few papers that investigate the full DMT of relay systems are [1], [5], and [6]. These references focus on half-duplex relaying and suggest that the system behavior is upper bounded by MISO or SIMO behavior.

Utilizing a multiple relay network as transmit or receive antenna array is beneficial; however, if we could configure the relay system like a MIMO, higher gains would be achievable. In [9] we showed that when we have multiple relays, maximum MIMO diversity is achievable, but this requires the relays to be clustered evenly around the source and the destination, meaning there should be a strong line of sight component between the clustered nodes.

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Despite these affirmative results about diversity gains, in terms of spatial multiplexing the relay systems behave quite different from MIMO. In [3], the authors compare a two-source two-destination cooperative system with a 2×2 MIMO and show that the former is multiplexing gain limited to 1, whereas the latter has maximum multiplexing gain of 2.

In [10], we studied the complete DMT of a single source-destination system with two relays for two different cases: Clustered and non-clustered. Assuming full-duplex operation, we showed that if the relay system is clustered it can achieve the 2×2 MIMO DMT up to multiplexing gain 1. However, if it is not clustered, the relay system can only behave like a 3×1 MISO or 1×3 SIMO. In conclusion, while clustering is necessary for higher diversity, even if the relay system is clustered and nodes are full-duplex, it can never fully mimic a physical MIMO system.

Most papers [1], [3], [5], [6] and [10] investigate single-antenna relay systems with one degree of freedom in the direct link. One wonders how the DMT would be affected if the degrees of freedom for the direct link was higher. In this work we allow 2 antennas on *both* the source and the destination. We consider a single relay that has 1 or 2 antennas. While the DMT of a full-duplex, single relay system with all the nodes having one antenna is very simple [10] and achieves 2×1 MISO upper bound for various relaying strategies such as decode-and-forward (DF) and compress-and-forward (CF), this is not the case when terminals have multiple antennas. We find DMT upper bounds for both clustered and non-clustered cases and investigate the performance of DF, partial decode-and-forward (PDF) and CF protocols. This problem is an initial step in extending the multi-relay DMT analysis of [10] to multi-antenna terminals.

Section II describes the general system model. Section III presents the system with single-antenna relay and in Section IV the relay has 2 antennas. Finally, in Section V we summarize our results.

II. System Model

We investigate the basic relay network with one source, one destination and one relay. The source and the destination have 2 antennas each, the relay has 1 or 2 antennas.

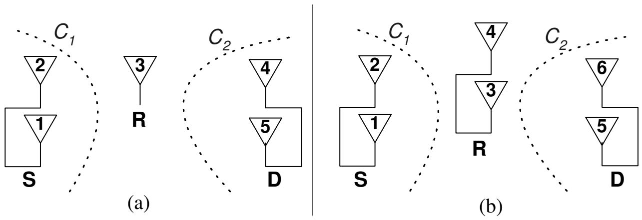

Fig. 1. Two-antenna source and destination system with (a) single-antenna relay, (b) two-antenna relay.

If the nodes are not clustered, we assume all inter-user channels have slow, frequency non-selective Rayleigh fading. However, if the nodes are close to each other, the Rayleigh assumption breaks due to the strong line of sight component in the received signal. We model this channel as an AWGN channel. In other words, if the nodes are closer to each other than the threshold distance d^*, then the channel is AWGN, otherwise it has Rayleigh fading. There is also a dead zone around the nodes, so the AWGN channel gain is bounded. Motivated by [9], we examine two cases. Fig. 2(a) shows the first case in which the nodes are in Rayleigh zones. We call this the non-clustered case. Similarly, Fig. 2(b) shows the clustered case with the nodes in their AWGN zones.

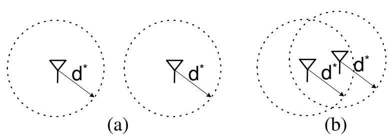

Fig. 2. (a) Non-clustered, (b) Clustered.

For both systems under investigation we number the antennas instead of the nodes with X_i denoting the signal antenna i transmits, Y_i the signal i^{th} antenna receives. If two antennas i and j, placed on different nodes, are in Rayleigh zones, then the channel gain in between is denoted as h_{ij}. All h_{ij} are independent for distinct i and j values and are complex Gaussian with zero mean. On the other hand, if the antennas are in AWGN zones, we denote the channel gain in between as the deterministic quantity $\sqrt{G_{ij}}$. The source and the relay have transmit SNR's SNR_S and SNR_R respectively. We assume they are on the same order, i.e. their ratio is a constant. There is also the effect of path loss but for fixed inter-node distances, the received SNR's are on the same order as well, and the DMT results are not affected. The noise terms at the relay and the destination are i.i.d. complex Gaussian with zero mean and variance 1.

Similar to [10], we assume the relay has channel state information (CSI) about all the channels in the system, including its incoming and outgoing channels as well as the channels in between the source and the destination. We will motivate why we need this CSI in Section III when we discuss the CF strategy. The source does not have instantaneous CSI. There is also short-term power constraint that has to be satisfied for each codeword transmitted. In addition, the relay is assumed to be full-duplex to obtain the best possible DMT. Furthermore, even for the system when the source, the destination and the relay have 1 antenna each, the best half-duplex relay strategy that achieves the DMT upper bound is unknown.

If $R(\mathrm{SNR})$ denotes the transmission rate of the system and $P_e(\mathrm{SNR})$ denotes the probability of error, we define multiplexing gain r and corresponding diversity $d(r)$ as

$$\lim_{\mathrm{SNR}\to\infty} \frac{R(\mathrm{SNR})}{\log \mathrm{SNR}} = r \quad \text{and} \quad \lim_{\mathrm{SNR}\to\infty} \frac{\log P_e(\mathrm{SNR})}{\log \mathrm{SNR}} = -d(r).$$

The DMT of an $m \times n$ MIMO is given by $d_{mn}(r)$, the best achievable diversity, which is a piecewise-linear function connecting the points $(k, d_{mn}(k))$, where $d_{mn}(k) = (m-k)(n-k)$, $k = 0, 1, ..., \min\{m, n\}$ [11]. Note that $d_{mn}(r) = d_{nm}(r)$.

III. System 1

The first system we study is the two-antenna source and destination, single-antenna relay system as shown in Figure 1(a).

A. Non-clustered Case

In [9], we prove that cut-set upper bounds provide DMT upper bounds as well. Therefore, we consider the cut-sets C_1 and C_2 and write

$$R \leq I(X_1 X_2; Y_3 Y_4 Y_5 | X_3) \tag{1}$$

$$R \leq I(X_1 X_2 X_3; Y_4 Y_5). \tag{2}$$

If there is no CSI at the source or the relay, we know the first cut-set behaves like 2×3 MIMO, and the second is like 3×2 both of which have the same DMT. If we allow CSI both at the source and the relay, under the short term power constraint, the best strategy to employ is beamforming among the transmitting antennas of the cut-set. For an m transmitter MIMO, with total transmit power P, the beamforming gain can at most be mP [11]. Therefore, CSI at the transmitters with no power allocation over time does not improve the DMT, it has the same the DMT when only receiver CSI is present. Thus, we conclude that $d_{23}(r) = d_{32}(r)$ is an upper bound to the system behavior as shown in Figure 3(a).

We next investigate how the cut-set upper bound can be achieved. In [10], we argued that for the non-clustered system,

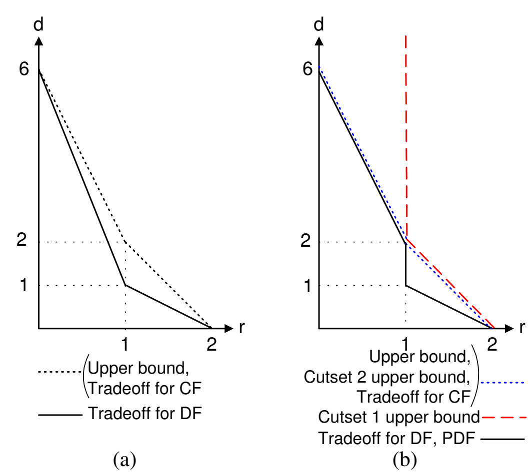

Fig. 3. DMT for System 1, (a) Non-clustered, (b) Clustered.

when each terminal has one antenna, both DF and CF achieve the cut-set upper bound, irrespective of the relay location. We now investigate the DF and CF achievable DMT for System 1.

1) DF: In the DF protocol, the source and the relay employ block Markov superposition coding. We assume fading stays constant for B blocks over which decoding is done. Hence, if the relay can decode, as the outage event dominates the error event, it will be able to decode for all B blocks. In this case the relay retransmits, otherwise, it will not transmit at all for all B. The relay can inform the destination whether it can decode or not at a negligible cost. With backward decoding, the destination can jointly decode the source and the relay signals. For the details of this protocol we refer the reader to [4], [10]. We can write the probability of outage at the destination as

$$\begin{aligned}P_{out} &= P(\text{outage}|\text{relay decodes})P(\text{relay decodes})\\ &+P(\text{outage}|\text{relay cannot decode})P(\text{relay cannot decode}).\end{aligned} \tag{3}$$

At high SNR at multiplexing gain r, this outage probability becomes

$$P_{out} \doteq \begin{cases} \frac{1}{\text{SNR}^{d_{32}(r)}}1+\frac{1}{\text{SNR}^{d_{22}(r)}}\frac{1}{\text{SNR}^{d_{21}(r)}} & \text{if} \quad 0\le r\le 1\\ \frac{1}{\text{SNR}^{d_{32}(r)}}0+\frac{1}{\text{SNR}^{d_{22}(r)}}1 & \text{if} \quad 1< r\le 2\end{cases}$$

If we expand the above expression, we find

$$\begin{aligned}P_{out} &\doteq \begin{cases} \frac{1}{\text{SNR}^{6-4r}}1+\frac{1}{\text{SNR}^{4-3r}}\frac{1}{\text{SNR}^{2(1-r)}} & \text{if} \quad 0\le r\le 1\\ \frac{1}{\text{SNR}^{4-2r}}0+\frac{1}{\text{SNR}^{2-r}}1 & \text{if} \quad 1< r\le 2\end{cases}\\ &\doteq \begin{cases} \frac{1}{\text{SNR}^{6-5r}} & \text{if} \quad 0\le r\le 1\\ \frac{1}{\text{SNR}^{2-r}} & \text{if} \quad 1< r\le 2\end{cases}.\end{aligned}$$

We observe that the DMT DF achieves is $d_{22}(r)+d_{21}(r)$, which is always less than the $d_{23}(r)$ upper bound. This is shown in Figure 3(a). The suboptimal behavior of DF arises because for multiplexing gains less than 1, the outage event when the relay cannot decode is dominant. In addition to this, for multiplexing gains larger than 1, the relay never participates in the communication because it is degrees of freedom limited and cannot decode large multiplexing gain signals. Note that for this region, we observe a shifted SISO behavior, as if we have turned off an antenna from each node. This observation will become clearer after we study the DF performance for 2 relay antennas in Section IV. Note that the relay does not use the full CSI in the above strategy. However, even if it did, and even with full CSI at the source, under the short term power constraint DMT of DF strategy does not change.

2) CF: As an alternative to decoding, the relay can perform compression. We assume the relay does Wyner-Ziv type compression assuming the source signal received at the destination as the side information. Thus, the relay utilizes its CSI, namely, both the source-destination and relay-destination channel gains to compute the accurate compression rate as illustrated in equation (5). Then

$$R_{CF}=I(X_1X_2;\hat{Y}_3Y_4Y_5|X_3) \tag{4}$$

is achievable when

$$I(\hat{Y}_3;Y_3|X_3Y_4Y_5)\le I(X_3;Y_4Y_5) \tag{5}$$

where the joint probability distribution is $p(x_1,x_2)p(x_3)$ $p(\hat{y}_3|x_3,y_3)p(y_3,y_4,y_5|x_1,x_2,x_3)$. The details of the proof can be found in [4].

Let $K_i=\det\left(\mathbf{I}+\text{SNR}\mathbf{H_i}\mathbf{H_i}^\dagger\right)$, $i=1,2,3$ where

$$\mathbf{H_1}=\begin{pmatrix} h_{14} & h_{24}\\ h_{15} & h_{25}\end{pmatrix},$$

$$\mathbf{H_2}=\begin{pmatrix} h_{13} & h_{23}\\ h_{14} & h_{24}\\ h_{15} & h_{25}\end{pmatrix},\quad \mathbf{H_3}=\begin{pmatrix} h_{14} & h_{24} & h_{34}\\ h_{15} & h_{25} & h_{35}\end{pmatrix}.$$

Assuming $\hat{Y}_3=Y_3+\hat{Z}_3$, $\hat{Z}_3$ is a complex Gaussian random variable with zero mean and variance $\hat{N}_3$ and independent from all other random variables, we have

$$\begin{aligned}I(\hat{Y}_3;Y_3|X_3Y_4Y_5) &= \log\left(1+\frac{K_2}{\hat{N}_3K_1}\right)\\ I(X_3;Y_4Y_5) &= \log\left(\frac{K_3}{K_1}\right).\end{aligned}$$

Then we can choose the compression noise variance $\hat{N}_3$ as $\hat{N}_3=K_2/(K_3-K_1)$. Thus

$$I(X_1X_2;\hat{Y}_3Y_4Y_5|X_3)=\log\left(\frac{K_2K_3}{K_2+K_3-K_1}\right) \tag{6}$$

is achievable. To prove the DMT of equation (6) we use the following upper bound

$$\begin{aligned}P_{out} &= P\left(I(X_1X_2;\hat{Y}_3Y_4Y_5|X_3)<r\log\text{SNR}\right)\\ &= P\left(\log\left(\frac{K_2K_3}{K_2+K_3-K_1}\right)<r\log\text{SNR}\right)\\ &\le P\left(\log\left(\frac{K_2K_3}{K_2+K_3}\right)<r\log\text{SNR}\right)\\ &\le P\left(K_2<2\text{SNR}^r\right)+P\left(K_3<2\text{SNR}^r\right)\\ &\doteq \frac{1}{\text{SNR}^{d_{23}(r)}}.\end{aligned}$$

This means the DMT for CF is $d(r)\ge d_{23}(r)$. On the other hand, the cut-set upper bound in Section III-A suggests $d(r)\le d_{23}(r)$. Thus, we conclude the CF protocol does not suffer any diversity-multiplexing gain loss and achieves the upper bound. This is because of the soft information transmission at the relay node. This observation also reveals that CF and DF protocols do not always behave similar, unlike the single antenna relay system. The degrees of freedom available also has an effect on relaying strategies.

B. Clustered Case

In [9], [10], we observed that for multiple relay systems, to maximize performance, clustering is essential. As we aim to extend our work to multiple relay systems with multi-antenna source-destination pairs, in this subsection we investigate the clustered system with a single relay. For the cut-set upper bound only the first bound $I(X_1X_2;Y_3Y_4Y_5|X_3)$ in equation (1) changes as the channel outputs Y_3, Y_4 and Y_5 are now related to the channel inputs X_1 and X_2 via the matrix $\mathbf{H_2'}$, which is

$$\mathbf{H_2'} = \begin{pmatrix} \sqrt{G} & \sqrt{G} \\ h_{14} & h_{24} \\ h_{15} & h_{25} \end{pmatrix}. \tag{7}$$

It is well known that a MIMO channel $\mathbf{H}$ can be considered as composed of its parallel eigen-channels with channel gains λ_i^2, where λ_i^2 are the eigenvalues of $\mathbf{H}\mathbf{H}^\dagger$. In [8], the authors argue that an eigen-channel is fully effective if $\mathrm{SNR}\lambda_i^2$ is of order SNR, and not effective if it is of order 1 or smaller. For a positive integer multiplexing gain r, the typical outage event is precisely r eigen-channels being fully effective and the rest ineffective, which is equivalent to $\mathbf{H}$ being rank r. Using this approach, we prove that the DMT of cut-set C_1 is $d(r) = \infty$ if $0 \leq r \leq 1$ and $d(r) = d_{23}(r)$ if $1 < r \leq 2$, which is shown in Figure 3(b). Note that this channel is not in outage for multiplexing gains less than 1. This is because the deterministic row of $\mathbf{H_2'}$ guarantees that the rank is at least 1, i.e. the Gaussian source-relay link is able to sustain a multiplexing gain of 1 with no outage. Nevertheless, this deterministic row does not improve the overall tradeoff for $r > 1$ and this cut-set still behaves like 2×3 MIMO. The proof is omitted due to limited space. The second cut-set is same as in Section III-A and has the tradeoff $d_{32}(r)$ for all r. Combining both cut-sets, the system is still limited to the 3×2 MIMO. We now study the achievable DMT of various relaying strategies.

1) DF: The DMT of the DF scheme, when the relay clustered with the source is shown in Figure 3(b). In this case, DF achieves the upper bound for $r < 1$. This is because the relay can always reliably decode the source information due to the Gaussian link in between. However, if $r > 1$, the relay can never decode the source reliably and the system reduces to the direct link, which exhibits a 2×2 MIMO behavior. As in the non-clustered case, relay CSI does not change the conclusions.

If the relay were clustered with the destination node, and the relay employed DF, the system behavior would be the same as the non-clustered case, because DF is beneficial if reliable decoding is possible with high probability.

2) PDF: One might think that partial decoding at the relay, which is known to improve the achievable rates with respect to DF [2], [4], might overcome the shortcomings of DF in DMT sense. Assume the source node constructs its signal as $\mathbf{X} = \mathbf{X_0} + \mathbf{X_1}$, $\mathbf{X_0}$ is directly sent to the destination and the relay only aims to decode and forward $\mathbf{X_1}$ by treating $\mathbf{X_0}$ as noise. Assume SNR_1 and SNR_0 power levels are assigned to $\mathbf{X_1}$ and $\mathbf{X_0}$ respectively and $\mathrm{SNR}_0 + \mathrm{SNR}_1 = \mathrm{SNR}_S$. Further, without loss of generality assume that $\mathrm{SNR}_0 \doteq \mathrm{SNR}^x$, $0 \leq x \leq 1$. If the relay node is clustered with the source, an upper bound for the achievable rates are given by

$$R_0 \leq \log\det\left(\mathbf{I_2} + \mathrm{SNR}_0 \mathbf{H_1}\mathbf{H_1}^\dagger\right) \tag{8}$$

$$R_1 \leq \log\left(1 + \frac{2G\mathrm{SNR}_1}{2G\mathrm{SNR}_0 + 1}\right) \tag{9}$$

$$R_0 + R_1 \leq \log\left(\det\left(\mathbf{I_2} + \mathrm{SNR}\mathbf{H_3}\mathbf{H_3}^\dagger\right)\right) \tag{10}$$

Equation (10) has the DMT $d_{32}(r)$ and the sum of (8) and (9) provides an upper bound to the DMT. This sum reveals that there will be no outage for multiplexing gains less than $1-x$, the diversity gain is infinity. This is because the term in equation (9) is deterministic. On the other hand, one can show that for multiplexing gains $1-x < r < 1+x$ we will observe $d_{22}((r-(1-x))/x)$. As the space is limited, we omit the proof. To maximize this upper bound, it is optimal to choose $x = 0$, i.e. to allocate all power to SNR_1, if $1 - x < r < 1$. On the other hand, if $1 < r < 1+x$, $x = 1$ is optimal, i.e. we should allocate all power to SNR_0 and not use the relay, but this leads to the same tradeoff DF achieves. Utilizing CSI at the source and the relay does not improve the DMT either. We conclude that PDF cannot bring additional benefits in terms of DMT. Using this result for the clustered case, one can also show that PDF does not improve the tradeoff for the non-clustered case either.

3) CF: In contrast to DF, or PDF, CF achieves the cut-set upper bounds with clustering as well. This is not surprising as we have proved that CF achieves the upper bound in the non-clustered case. The proof follows the same lines with the previous section with K_2 replaced with $K_2' = \det\left(\mathbf{I_3} + \mathrm{SNR}\mathbf{H_2'}\mathbf{H'}_2^\dagger\right)$.

The above observations point to the fundamental limitations of the DF protocol. If the relay does DF (or PDF), for multiplexing gains larger than 1, the relay is not functional as it is not capable of processing $r \geq 1$. For $r < 1$, if the relay is not clustered, the outage event when the relay cannot decode is dominant, which results in a suboptimal DMT. This suggests that not only the geography of nodes [4], but also the degrees of freedom available affects the relaying strategies. The DF limitations are further emphasized in the following section.

IV. SYSTEM 2

We now consider the system in Figure 1(b).

A. Non-clustered Case

The cut-set bounds are very similar to equations (1) and (2) and result in DMT equal to $d_{24}(r)$ and $d_{42}(r)$ respectively.

1) DF: To find the DF achievable tradeoff, we again use equation (3). Following the same analysis as in Section III-A, we find that DF achieves $d(r) = d_{22}(r) + d_{22}(r)$, $0 \leq r \leq 2$ as shown in Figure 4(a).

Similar to our observations in Section III, the event that the relay cannot decode becomes dominant in the outage expression. Note that this is despite the relay has the same number of antennas as the source and the destination. For $1 < r < 2$, DF behaves as if an antenna is removed from all nodes, resulting in a shifted DMT of $d_{21}(r-1)$. The reason is the inherent MIMO behavior in the source-relay and source-destination channels. Consider an $m \times n$ MIMO system with no relays. For multiplexing gains $k \leq r \leq \min\{m, n\}$, the DMT is that of a shifted $(m-k) \times (n-k)$ MIMO. In other words, it is as if we dedicate a pair of source-destination antennas for each additional multiplexing gain of 1. In a MIMO system

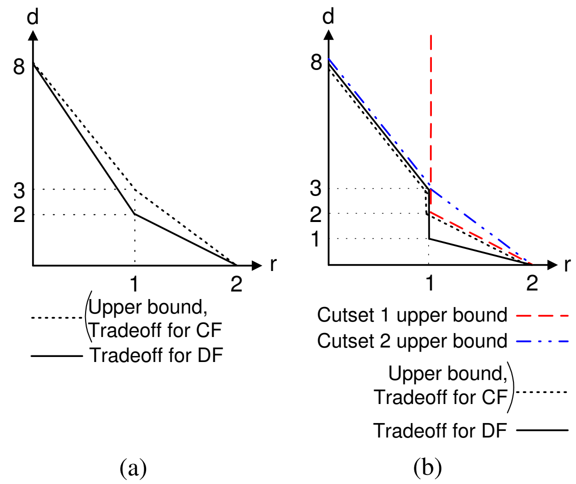

Fig. 4. DMT for System 2, (a) Non-clustered, (b) Clustered.

in which relay(s) use DF strategy, the same would happen between the source and the relay(s) as well as the direct link from the source to the destination. Therefore, combining with Section III-A, we conclude that for multi-antenna relay systems DF protocol is suboptimal.

2) CF: The CF strategy, in which the relay uses vector Wyner-Ziv compression for (Y_3, Y_4), achieves the DMT upper bound in this case as well. Due to space limitations, we do not provide the proof here.

B. Clustered Case

For single antenna systems with multiple relays [10] and System 1, we observed that clustering improves achievable DMT. Interestingly, this is not the case for all multiplexing gains for System 2. The first cut-set bound in equation (1) has the channel gain matrix $\mathbf{H_4'}$

$$\mathbf{H_4'} = \begin{pmatrix} \sqrt{G} & \sqrt{G} \\ \sqrt{G} & \sqrt{G} \\ h_{15} & h_{25} \\ h_{16} & h_{26} \end{pmatrix}$$

which adds an extra row to $\mathbf{H_2'}$ that is linearly dependent on the first row of $\mathbf{H_2'}$. This does not change the rank and the first cut-set bound is the same as in Section III-B, that is even though the relay has an extra antenna than System 1, cut-set bound C_1 cannot utilize this. By moving the relay to the AWGN zone of the source, we actually decreased the degrees of freedom in the source-relay channel. The AWGN channel in between only introduces power gain but not degrees of freedom. The second cut-set still behaves like a 4×2 MIMO as in the non-clustered case. The cut-set bounds are shown in Figure 4(b). Overall, with clustering we have the same DMT upper bound as the non-clustered case for $r \leq 1$, but for $1 < r \leq 2$ range, clustering *decreases* the DMT upper bound.

Similar to Section III-B, we can prove that clustering is essential for DF to achieve the DMT upper bound for $r \leq 1$. For $r > 1$, although both the source and the relay have 2 antennas each, the relay can never decode the source reliably as the AWGN channel between the source and the relay can only sustain a multiplexing gain up to 1. Therefore, clustering in this case *degrades* the DF performance. However, the CF strategy still achieves the upper bound.

V. Conclusion

In this work we study a full-duplex multi-antenna relay system DMT and investigate the effects of increased degrees of freedom on relaying strategies DF, PDF and CF. We also examine the effect of clustering on both the DMT upper bounds and achievability results. We compare a single-antenna relay system with a 2-antenna relay when both the source and the destination have 2 antennas.

We find that multi-antenna relay systems have fundamental differences from their single-antenna counterparts. Increased degrees of freedom affects the DMT upper bounds and the performance of different relaying strategies leading to some counterintuitive results. Although the DF protocol is simple and effective to achieve the DMT upper bounds in single antenna relay systems, it proves to be suboptimal for multi-antenna relay systems, even if the relay has the same number of antennas as the source. The PDF protocol does not improve the DF achievable DMT either. On the other hand, the CF strategy is highly robust and achieves the DMT upper bounds for all multiplexing gain values.

In addition to these, clustering is essential for DF to achieve the DMT upper bound in $0 \leq r \leq 1$ the range, but does not help in $1 < r \leq 2$, neither for 1 nor for 2 antenna relays. What's more, it has an adverse effect on both the upper bound and the DF achievable DMT if the relay has multiple antennas due to decreased degrees of freedom in the source-relay channel.

As future work, we plan to extend the DMT analysis to multi-antenna multiple relay systems in a manner we did in [10].

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