Diversity-Multiplexing Tradeoff for the MIMO Static Half-Duplex Relay

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***Abstract*—In this work, we investigate the diversity-multiplexing tradeoff (DMT) of the multiple-antenna (MIMO) static half-duplex relay channel. The relay channel is assumed to be symmetric in terms of number of antennas, i.e. the source and the destination have n antennas each, and the relay has m antennas. A general expression is derived for the DMT upper bound, which can be achieved by a compress-and-forward protocol at the relay, under certain assumptions. It is observed that the static half-duplex DMT matches the full-duplex DMT when the relay has a single antenna, and is strictly below the full-duplex DMT when the relay has multiple antennas.**

I. Introduction

Next generation wireless communication will take place over large networks with multiple-antenna (MIMO) terminals. To be able to understand how to operate these complex networks optimally, we need a solid theoretical background. The diversity-multiplexing tradeoff (DMT) [1] provides such a theoretical background for multiple-antenna systems. It establishes the fundamental tradeoff between reliability and rate via diversity and multiplexing gains. Diversity gain is a measure of reliability, and shows how fast the error probability decays with increasing signal-to-noise ratio (SNR). Similarly, multiplexing gain is related to the transmission rate of the system, and shows how this rate increases with increasing SNR.

Cooperation/relaying is going to be fully integrated in the standard operation of next generation wireless communications [2]. In wireless channels, nearby nodes can overhear source messages for free. Due to this *wireless broadcast advantage*, relays can process the overheard information and forward it to the destination terminal. The destination can then combine the direct signal from the source and the forwarded signals from the relays to improve the system performance [3], [4], [5].

To understand large, multiple-antenna cooperative networks, the theory has to account for practical system constraints such as power, bandwidth or delay limitations. Another important constraint in relay channels is the half-duplex constraint. As the transmit power overwhelms the received power, wireless devices cannot transmit and receive at the same time in the same band. In other words, they are not full-duplex; they have to operate in half-duplex mode.

The half-duplex relay channel DMT has been already studied in [5], [6], [7], [8], [9], [10], and [11]. The first three of these papers consider the single-antenna single-relay channel, and respectively study the orthogonal amplify-and-forward and decode-and-forward DMT, the DMT of the non-orthogonal amplify-and-forward and the dynamic decode-and-forward protocols, and finally a new "quantize-and-map" relaying scheme that achieves the half-duplex DMT. In [8], Prasad and Varanasi investigate a multiple relay channel with single antenna source and relays and a multiple-antenna destination and propose space-time coding strategies. In [9], the authors suggest relay selection as an alternate way to obtain DMT improvements in a multiple relay setting. The paper [10] extends the non-orthogonal amplify-and-forward protocol to the multiple-antenna multiple-relay setting. In [11], the authors study the multiple-antenna single-relay channel DMT. They first state the static half-duplex DMT upper bound without explicitly computing it, and then show that the compress-and-forward protocol achieves this upper bound for any number of antennas at the source, the relay or the destination.

The above mentioned papers enlighten many important issues about the DMT of the half-duplex relay channel. However, while compress-and-forward is known to be DMT achieving (under suitable assumptions on the available channel state information), the exact static half-duplex DMT for arbitrary antenna configurations has not been computed yet. In this paper, we address this problem and compute the static half-duplex DMT. We consider the symmetric relay channel with n antennas at the source and the destination, and m antennas at the relay.

A natural question regarding the half-duplex constraint on the relay is whether this constraint imposes a limitation on the system performance, in comparison to a relay operating in full-duplex mode. Our DMT computation leads to the following conclusion: for half-duplex and full-duplex DMT to be equal, the number of relay antennas has to equal to 1. If the relay has two or more antennas, the half-duplex constraint imposes a strict limitation on the system performance, *irrespective* of n, the number of antennas at the source and the destination. The only case where the half-duplex constraint imposes no limitation (in the DMT sense) is the case where the relay has a single antenna. It is surprising to observe that when the relay

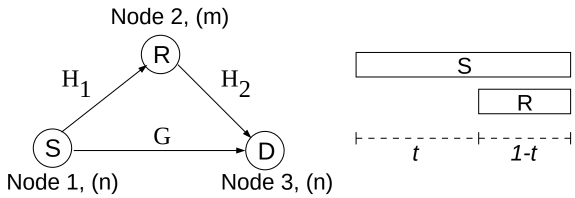

Fig. 1. The half-duplex relay channel. The source and the destination have n antennas each and the relay has m antennas. The relay listens for t fraction of the time, and transmits in the remaining $1-t$.

has multiple antennas, increasing the number of antennas at both the source and the destination does not allow to get rid of the half-duplex limitation.

The paper is structured as follows. In Section II we introduce the system model. In Section III we present the cut-set upper bound on the DMT for a half-duplex relay channel. In Section IV, we present our computation method for the DMT upper bound. In Section V, we give the explicit expression of the DMT for any values of m and n. Finally, in Section VI, we conclude.

II. System Model

We study the MIMO static half-duplex relay channel with n antennas at the source and the destination, and m antennas at the relay, see Fig. 1. The relay listens for a fraction t of the time, and transmits in the remaining $(1-t)$ fraction, $t \in [0,1]$.

When the relay is listening, i.e. the system is in state q_1, the received signals at the relay and the destination are

$$Y_{2,1} = H_1 X_{1,1} + Z_{2,1}, \quad Y_{3,1} = G X_{1,1} + Z_{3,1} \tag{1}$$

When the relay is transmitting, i.e. the system is in state q_2, the received signal at the destination is

$$Y_{3,2} = G X_{1,2} + H_2 X_{2,2} + Z_{3,2} \tag{2}$$

The column vectors $X_{1,l}$ and $X_{2,l}$, $l = 1,2$ are respectively of length n and m and denote the signals transmitted by the source and the relay in state q_l. As the relay is half-duplex, $X_{2,1} = 0$. Similarly $Y_{2,l}$ and $Y_{3,l}$, $l = 1,2$ are respectively of length m and n and denote the received signals at the destination and at the relay. Notice that $Y_{2,2} = 0$. The channel gain matrices G, H_1, and H_2 are of size $n \times n$, $m \times n$, and $n \times m$ respectively and are assumed to have independent and identically distributed (i.i.d.) zero mean complex Gaussian entries with unit variance. Fading is supposed slow and frequency non-selective, so that it remains fixed for one frame length. The relay and destination noise vectors $Z_{2,1}$ and $Z_{3,l}$ are of length m and n with i.i.d. complex Gaussian entries with zero mean and unit variance.

The source and the relay have average short-term power constraints P_1 and P_2 for each codeword transmitted. As a constant scaling in transmitted power does not change the DMT [1], we will assume that $P_1 = P_2 = P/2$ in the rest of the paper.

There is no transmitter channel state information at the source or at the relay. On the other hand, we assume that there are pilot signals to measure receiver channel gains, and thus the relay and the destination know their incoming fading channel matrices.

In this paper, we compute the static half-duplex DMT upper bound, when there is only receiver channel state information. On the other hand, the bound presented in the following is still valid if transmitter channel state information is available at the source, and the relay and the destination know all the channel gains in the system, but the system operates at *constant information rate*, under short-term power constraint. Under these assumptions, the compress-and-forward scheme achieves the DMT upper bound [11].

Although the relay is informed about its incoming channel gains, we assume that the relay does not use this information to determine the amount of time it listens or transmits. We therefore assume that the relay employs a *static protocol* for communication, in which the decision of transmitting or receiving is not based on the channel matrix realizations, but only on their respective distributions. *Dynamic protocols*, in which the relay chooses t based on its channel state observations, have the potential to achieve better DMT. However, dynamic protocols are more complex to study and the half-duplex DMT computation already constitutes a hard problem. In this paper, we thus focus on static protocols only.

Note that in (1)-(2), we assumed fixed relaying. In fixed protocols, the relay does not carry information via breaking its transmission and reception intervals into smaller blocks and controlling its state variable. If the relay protocol were not fixed but random, the state variable could be used to convey additional information. However, the increase is at most one bit and the DMT results remain the same for random protocols [11].

As the source node does not have channel state information, an outage occurs if the mutual information at the destination is not large enough to support the fixed target communication rate the source chooses. In this paper, we study the minimum outage probability achieved by such a system at high SNR, following the approach of Zheng and Tse in [1]. It is assumed that the reader is familiar with the DMT definition [1], in which d denotes diversity and r is the multiplexing gain.

In the next section, we present the cut-set bound mutual information expressions for the MIMO static half-duplex relay channel, which we will use to establish the DMT upper bound.

III. The DMT Upper Bound for the Relay Channel

In [11], it is shown that cut-set bounds and the associated outage expressions can be used to find DMT upper bounds. Following the same approach, we provide next the cut-set bounds for the static half-duplex relay channel and the corresponding outage probability expressions to find the half-duplex relay channel DMT for static protocols.

In the half-duplex, fixed and static relay channel, the mutual information expressions for the cut-sets around the source and the destination are respectively equal to

$$I_S(t) = tI(X_1; Y_2 Y_3 | q_1) + (1-t) I(X_1; Y_3 | X_2, q_2) \tag{3}$$

$$I_D(t) = tI(X_1; Y_3 | q_1) + (1-t) I(X_1 X_2; Y_3 | q_2) \tag{4}$$

To find the best static DMT upper bound, we need to find the maximum of these mutual information expressions, which are obtained when Gaussian codebooks are used and the input covariance matrix is chosen optimally [11]. We can further upper bound (3) and (4) as $I_S(t) \leq I'_S(t)$ and $I_D(t) \leq I'_D(t)$ [11],

$$\begin{aligned} I'_S(t) &= t \log\det\left(I_{m+n} + P \begin{bmatrix} G \\ H_1 \end{bmatrix} \begin{bmatrix} G \\ H_1 \end{bmatrix}^*\right) \\ &\quad + (1-t)\log\det\left(I_n + PGG^*\right) \\ I'_D(t) &= t\log\det\left(I_n + PGG^*\right) \\ &\quad + (1-t)\log\det\left(I_n + P[G, H_2]\,[G, H_2]^*\right) \end{aligned}$$

where I_n denotes the identity matrix of size $n \times n$, and * denotes conjugate transpose. Thus, the outage probability corresponding to a target rate $r \log P$ is lower bounded by

$$\begin{aligned} \mathbb{P}_{\text{out}}(r\log P) &\geq \min_{t\in[0,1]} \max\{\mathbb{P}(I'_S(t) < r\log P), \\ &\qquad \mathbb{P}(I'_D(t) < r\log P)\} \\ &\triangleq \mathbb{P}_{\text{out},0}(r\log P) \end{aligned} \tag{5}$$

These expressions lead to the DMT upper bound

$$d_{\text{HD}}(r) \leq \max_{t\in[0,1]} \min\{d_S(r,t), d_D(r,t)\} \triangleq d_{\text{HD},0}(r) \tag{6}$$

where $\mathbb{P}_{\text{out}}(r\log P) \doteq P^{-d_{\text{HD}}(r)}$, $\mathbb{P}(I'_S(t) < r\log P) \doteq P^{-d_S(r,t)}$, $\mathbb{P}(I'_D(t) < r\log P) \doteq P^{-d_D(r,t)}$ and $\mathbb{P}_{\text{out},0}(r\log P) \doteq P^{-d_{\text{HD},0}(r)}$ [1].

When the relay is half-duplex, it can only transmit during a fraction $t \in [0,1]$ of the time (and therefore receive during the other fraction $1-t$). Because of the static protocol assumption, the fraction t is a fixed number, chosen according to the distribution of the channel coefficients only, and not to their realizations. Thus, the symmetry of the problem implies that the optimal value of (5) and thus of (6) is reached when $t = 1/2$. We therefore have

$$\mathbb{P}_{\text{out},0}(r\log P) = \mathbb{P}\left(\hat{I}_D < r\log P\right) \tag{7}$$

where $\hat{I}_D =$

$$\frac{1}{2}\log\det\left(I_n + PGG^*\right) + \frac{1}{2}\log\det\left(I_n + P[G,H]\,[G,H]^*\right)$$

and H is an $n \times m$ matrix with the same distribution as H_2. The above outage probability in (7) is hard to calculate as the Hermitian matrices GG^* and $[G,H]\,[G,H]^*$ are correlated. Assuming different number of antennas at all nodes complicates the problem even more. Therefore, in this paper we assume same number of antennas at both the source and the destination. Thus G is $n \times n$.

In Sections IV and V, we compute explicitly the diversity order of the above outage probability, $d_{\text{HD},0}(r)$, and compare it to the full-duplex relay channel DMT [11]. When the relay is full-duplex, it forms a virtual antenna array with either the transmitter or the receiver and the performance of the system is upper bounded by that of an $(n+m) \times n$ MIMO point-to-point channel. In the following, we will use this performance as a benchmark and exhibit what loss is to be expected when operating the relay in half-duplex mode.

In this paper our focus is on the computation of the DMT itself. Note that the bound we compute is achievable by the compress-and-forward protocol [11]. This protocol is the only one known to achieve the half-duplex DMT, when both the source and the destination have multiple antennas. We refer the reader to [3] and [11] for the details of the protocol and its DMT optimality proof.

IV. Computation Method

In the following, our aim is to compute the diversity order of the outage probability (7). Notice that

$$\begin{aligned} \hat{I}_D &= \log\det(I_n + PGG^*) \\ &\quad + \frac{1}{2}\log\det(I_m + PH^*(I_n + PGG^*)^{-1}H) \end{aligned}$$

Since G and H are independent and their respective distributions are unitarily invariant, the above random variable has the same distribution as

$$\log\det(I_n + P\Lambda) + \frac{1}{2}\log\det(I_m + PH^*(I_n + P\Lambda)^{-1}H)$$

where $\Lambda = \text{diag}(\lambda_1, \ldots, \lambda_n)$ and $\lambda_1 \geq \ldots \geq \lambda_n$ are the eigenvalues of GG^*. Let now $B = (I_n + P\Lambda)^{-1/2}H$ ($n \times m$ matrix). *Conditioned on* Λ (which is independent of H), the joint distribution of the entries of B is given by

$$p(B|\Lambda) = \frac{1}{\pi^{nm}} \det(I_n + P\Lambda)^m \exp(-\text{Tr}(B^*(I_n + P\Lambda)B))$$

The spectral decomposition of the $n \times n$ matrix BB^* reads: $BB^* = UMU^*$, where U is a $n \times (n \wedge m)$ ($n \wedge m \triangleq \min\{n,m\}$) matrix such that $UU^* = I_n$ and $M = \text{diag}(\mu_1, \ldots, \mu_{n\wedge m})$. The Jacobian of the transformation $B \mapsto (M,U)$ is given by

$$J(M,U) = (\det M)^{|m-n|}\,\Delta(M)^2,$$

where $\Delta(M) = \prod_{j,k=1,j<k}^{n\wedge m}(\mu_j - \mu_k)$. Therefore, conditioned on Λ, the joint distribution of (M,U) is given by

$$\begin{aligned} p(M,U|\Lambda) &= C_{n,m}\det(I_n + P\Lambda)^m \\ &\quad \times \exp(-\text{Tr}(UMU^*(I_n + P\Lambda)))\,(\det M)^{|m-n|}\,\Delta(M)^2 \end{aligned}$$

so U is independent of M and distributed according to the Haar measure on the set

$$\mathcal{U}(n,m) = \{\text{complex } n \times (n\wedge m) \text{ matrices } U \,:\, UU^* = I_n\}$$

that is, the columns of U form a set of $n \wedge m$ orthonormal vectors, which are uniformly distributed on the sphere $\{z \in \mathbb{C}^n : |z| = 1\}$.

This allows us to compute the conditional distribution $p(M|\Lambda)$:

$$\begin{aligned} p(M|\Lambda) &= \int_{\mathcal{U}(n,m)} dU\, p(M,U|\Lambda) \\ &= C_{n,m}\det(I_n + P\Lambda)^m\,(\det M)^{|m-n|}\,\Delta(M)^2 \\ &\quad \times \exp(-\text{Tr}(M)) \int_{\mathcal{U}(n,m)} dU\,\exp(-P\text{Tr}(UMU^*\Lambda)) \end{aligned}$$

[1] Note that $f(P) \doteq P^{-c}$ means $\lim_{P\to\infty} \log f(P)/\log P = c$. Inequalities are defined similarly.

Let finally $\Delta(\Lambda) = \prod_{j,k=1,j<k}^{n}(\lambda_j - \lambda_k)$. It is a well known fact that $p(\Lambda) = C_n\, \Delta(\Lambda)^2\, \exp(-\mathrm{Tr}(\Lambda))$, the classical Wishart distribution, so we obtain

$$\begin{aligned} & p(\Lambda, M) = p(\Lambda)\, p(M|\Lambda) \\ & = C_{n,m}\, \det(I_n + P\Lambda)^m\, (\det M)^{|m-n|}\, \Delta(\Lambda)^2\, \Delta(M)^2 \\ & \times \exp(-\mathrm{Tr}(\Lambda + M)) \int_{\mathcal{U}(n,m)} dU\, \exp(-P\mathrm{Tr}(UMU^*\Lambda)) \end{aligned} \tag{8}$$

Notice that the spherical integral on the far right may be rewritten as

$$\int_{\mathcal{U}(n,m)} dU\, \exp(-P\mathrm{Tr}(UMU^*\Lambda)) = \mathbb{E}_U\left(\exp\left(-P\sum_{j,k=1}^{n,n\wedge m} \lambda_j\, \mu_k\, |u_{jk}|^2\right)\right)$$

where $\mathbb{E}_U$ denotes the expectation with respect to the Haar measure on $\mathcal{U}(n,m)$.

Now, (7) can be rewritten as

$$\mathbb{P}_{\mathrm{out},0}(r\log P) = \int_{\substack{\{\Lambda, M\,:\,\log\det(I_n+P\Lambda)\\ +\frac{1}{2}\log\det(I_n+PM)<r\log P\}}} p(\Lambda, M)\, d\Lambda\, dM$$

Following Zheng and Tse, let us make the change of variables $\lambda_j = P^{-\alpha_j}$ and $\mu_k = P^{-\beta_k}$, where $\alpha_1 \leq \ldots \leq \alpha_n$ and $\beta_1 \leq \ldots \leq \beta_{n\wedge m}$. The behaviour in the limit $P \to \infty$ of most terms in (8) is well known, except for the spherical integral. Let us define

$$\mathcal{I}(P) = \mathbb{E}_U\left(\exp\left(-\sum_{j,k=1}^{n,n\wedge m} P^{1-\alpha_j-\beta_k}\, |u_{jk}|^2\right)\right)$$

The asymptotic behaviour of $\mathcal{I}(P)$ is given by

$$\lim_{P\to\infty} \frac{\log(\mathcal{I}(P))}{\log P} = \begin{cases} -\infty, & \text{if } \max_{1\leq j\leq m\wedge n}(1-\alpha_{n+1-j}-\beta_j) > 0 \\ -\sum\limits_{\substack{j,k=1\\ j+k\leq n}}^{n,n\wedge m} (1-\alpha_j-\beta_k)^+, & \text{otherwise} \end{cases} \tag{9}$$

As space is limited, we refer the reader to [12] for the proof of this asymptotic equality. Notice that an alternate computation technique for spherical integrals was already developed in [10], in a slightly different context.

The diversity order corresponding to the outage probability (7) can now be computed via the standard method developed in [1]. The computation leads to the following optimization problem:

$$\begin{aligned} & d_{\mathrm{HD},0}(r) \\ = \quad & \min \sum_{j=1}^{n} (2n-2j+1)\,\alpha_j + \sum_{j=1}^{n\wedge m} (n+m-2j+1)\,\beta_j \\ & -m\sum_{j=1}^{n}(1-\alpha_j)^+ + \sum_{\substack{j,k=1\\ j+k\leq n}}^{n,m\wedge n} (1-\alpha_j-\beta_k)^+ \end{aligned} \tag{10}$$

where the minimization takes place over the set of variables $\alpha_n \geq \ldots \geq \alpha_1 \geq 0$ and $\beta_{m\wedge n} \geq \ldots \geq \beta_1 \geq 0$ such that

$$\sum_{j=1}^{n}(1-\alpha_j)^+ + \frac{1}{2}\sum_{j=1}^{m\wedge n}(1-\beta_j)^+ \leq r$$

and $\alpha_{n+1-j} + \beta_j \geq 1$, for all $j \in \{1, \ldots, m\wedge n\}$. In the next section, we present the solution of the above minimization problem.

V. Explicit Expression for the Diversity-Multiplexing Tradeoff

The minimization problem (10) constitutes a convex programming problem. It therefore has a unique solution, which is given below.

Let us define l_0 to be the minimum of n and $\lfloor \frac{m+1}{3} \rfloor$ (so $l_0 = n$ if $m \geq 3n-1$; otherwise, l_0 is an integer number between 0 and $n-1$). It turns out that this number delimitates three different regimes for the DMT curve:

a) For $0 \leq r \leq l_0/2$ (low multiplexing gain), the outage probability of the half-duplex relay is determined by the outage of the $n \times m$ relay link matrix H (the direct link matrix G being already off). In this regime, the corner points of the diversity curve are given by

$$d_{\mathrm{HD},0}(l/2) = n^2 + (m-l)(n-l), \quad l \in \{0, \ldots, l_0\}$$

The main outage event is the event that only l relay links are active, while all n direct links are off. More precisely, we have

$$\begin{aligned} \alpha_1^* &= \ldots = \alpha_n^* = 1 \\ \beta_1^* &= \ldots = \beta_l^* = 0,\ \beta_{l+1}^* = \ldots = \beta_{m\wedge n}^* = 1 \end{aligned}$$

b) For $l_0/2 \leq r \leq n - l_0/2$ (intermediate multiplexing gain; notice that this regime does not exist if $l_0 = n$, that is, if $m \geq 3n-1$), the outage probability is determined by a combination of direct link G and relay link H outages, and the corner points of the diversity curve are given by

$$d_{\mathrm{HD},0}(l_0/2+l) = l_0^2 + (n+m-l)(n-l_0-l),\ l \in \{0, \ldots, n-l_0\}$$

The main outage event is that only l direct links and $l_0 + l$ relay links are active. More precisely, we have

$$\begin{aligned} \alpha_1^* &= \ldots = \alpha_l^* = 0,\ \alpha_{l+1}^* = \ldots = \alpha_n^* = 1 \\ \beta_1^* &= \ldots = \beta_{l_0}^* = 0,\ \beta_{l_0+1}^* = \ldots = \beta_{m\wedge n}^* = 1 \end{aligned}$$

c) For $n - l_0/2 \leq r \leq n$ (high multiplexing gain), the outage probability is determined by the outage of the $n \times n$ direct link matrix G only, and the corner points of the diversity curve are given by

$$d_{\mathrm{HD},0}(n - l/2) = l^2, \quad l \in \{0, \ldots, l_0\}$$

The main outage event is that only $n-l$ direct links are active and all n relay links are active. More precisely, we have

$$\begin{aligned} \alpha_1^* &= \ldots = \alpha_{n-l}^* = 0,\ \alpha_{n-l+1}^* = \ldots = \alpha_n^* = 1 \\ \beta_1^* &= \ldots = \beta_l^* = 0,\ \beta_{l+1}^* = \ldots = \beta_{m\wedge n}^* = 1 \end{aligned}$$

Finally, the diversity curve $d_{\mathrm{HD},0}(r)$ is the (convex) piecewise linear curve interpolating between all these corner points.

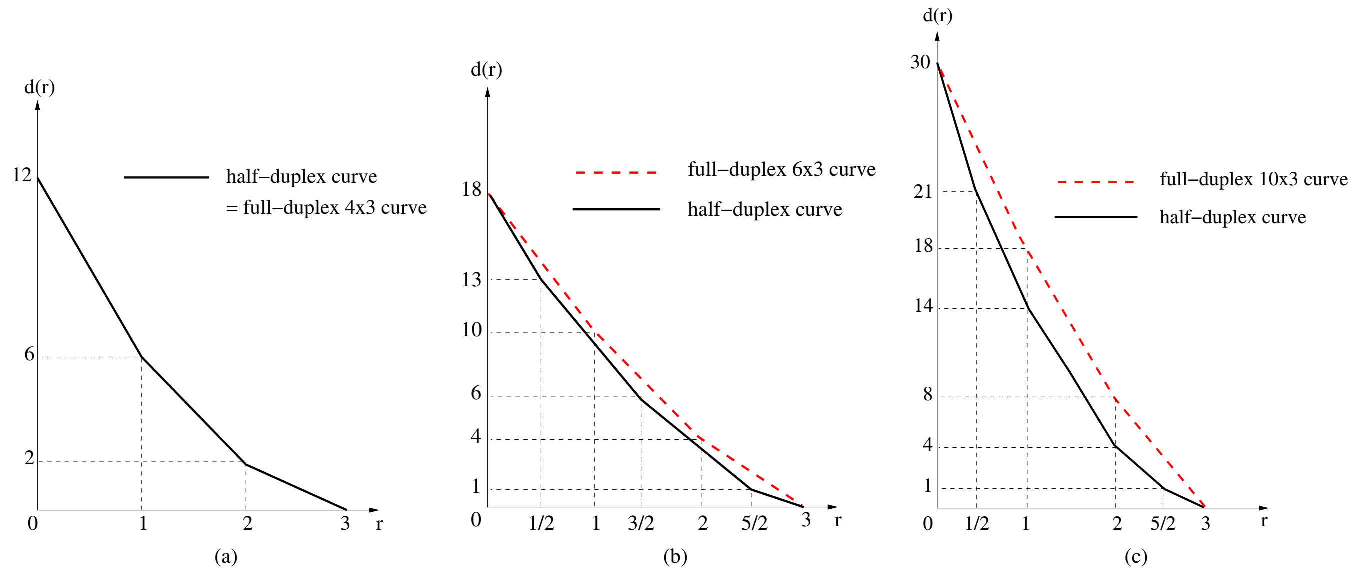

Fig. 2. Half-duplex DMT curves for $n = 3$ and (a) $m = 1$, (b) $m = 3$, (c) $m = 7$.

One has to be careful relating the non-zero α_j and β_j values to the number of active links. For the direct channel matrix G, the number of active links is equal to the number of non-vanishing singular values $\sqrt{\lambda_j}$ of G, that is, the number of α_j being equal to zero (since $\lambda_j = P^{-\alpha_j}$). For the relay channel matrix H, such a relation is less immediate, since the numbers β_j are related to the singular values $\sqrt{\mu_j}$ of the *combined* channel matrix $(I_n + PGG^*)^{-1/2}H$ (through the equality $\mu_j = P^{-\beta_j}$). From this relation, we deduce that the number of active links in the relay channel matrix H is equal to the *sum* of the number of α_j and β_j being equal to zero.

The DMT curves are shown for the case where the source and the destination have $n = 3$ antennas each and the relay has $m = 1$, 3 and 7 antennas in Fig. 2. As expected, the loss due to the half-duplex constraint increases as m gets larger.

Notice that, as already mentioned in the introduction, the only case where the half-duplex curve matches the full-duplex $(m+n)\times n$ curve is when $l_0 = 0$, i.e. $m = 1$, and n takes any integer value. For any $m \geq 2$, the half-duplex curve does not match (and is therefore strictly below) the full-duplex curve. This can be readily checked by noticing that the horizontal positions of the corner points do not match: in the half-duplex case (for $m \geq 2$), some corner points are located at non-integer multiplexing gains.

VI. Conclusion and Perspectives

In this work, we have computed the half-duplex relay channel diversity-multiplexing tradeoff (DMT) in the symmetric case, where the source and the destination have n antennas each and the relay has m antennas. Static relaying protocols were considered. Our results show that the static half-duplex DMT is equal to the full-duplex DMT only when the relay has a single antenna. In all other cases, performance losses are to be expected because of the half-duplex constraint. The problem is more complex and left open for general antenna configurations and for dynamic half-duplex relay operation.

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