

# Broadcast Strategies for the Fading Relay Channel

Melda Yuksel and Elza Erkip

Department of Electrical and Computer Engineering

Polytechnic University

Brooklyn, New York 11201-3840

Contact e-mail: myukse01@utopia.poly.edu

**Abstract**—Broadcast strategy is an encoding technique that enables the reliably decoded rate to adapt to the actual channel state. This can be achieved if the transmitted signal is composed of superimposed information for different fading levels. In this work we apply the broadcast strategy to fading relay channels. The source encodes its information to two levels, the first one is decoded if the channel state is “bad”, and the superimposed information is decoded after the first one, if the channel state is “good”. This approach allows decoding some partial information both at the relay and at the destination, increasing the overall throughput of the system compared to amplify and forward (AF) and modified decode and forward (MDF), while resulting in two levels of diversity. We also argue that the broadcast approach over the relay channel also results in lower overall distortion when the source is also taken into account.

## I. INTRODUCTION

In wireless communications because of fading the received signal can be severely degraded. In order to overcome fading, the standard diversity techniques are time, frequency and spatial diversity [6]. Spatial diversity is typically envisioned as having multiple transmit and/or receive antennas. Yet, another form of spatial diversity is *user cooperation diversity* or simply *cooperative diversity*, which utilizes wireless relays to form a virtual antenna array [7], [8]. Cooperation can also be used to provide higher rates and results in a more robust system. Recently proposed simple cooperation schemes, which take into account the practical constraint that the relay cannot transmit and receive at the same time, include amplify and forward (AF), decode and forward (DF) and their adaptive versions such as incremental relaying [3].

Amplify and forward (AF) is a simple scheme where the relay scales the amplitude of the signal it received to its own transmission power and then transmits to the destination. However, this is an analog method that results in noise amplification. On the other hand, in the DF protocol, the relay can help the destination only if it can decode the source signal reliably, otherwise the relay does not send any information to the destination. As a result, this method does not make full use of all the resources available in the system, and hence causes some loss in performance. In this work we propose a new wireless relaying strategy that uses Cover-type broadcast ideas [1] to eliminate both noise amplification and information loss at the relay.

When studying slow fading channels, the concept of capacity versus outage is useful [5]. An outage event occurs when the instantaneous channel realization is not good enough to support reliable communication for the fixed rate the transmitter operates at. However, this is a discrete approach to communication over fading channels: The system can either operate at the a-priori chosen fixed rate or not. The broadcast strategy introduced in [9] enables the rate of information to depend on the instantaneous channel realization even if the transmitter has no channel state information. This is achieved through a broadcast code, which is composed of superimposed information at different rates. This way the received transmission rate can adapt to the actual channel conditions without the need of a feedback link to the transmitter. This property of the broadcast strategy also makes it useful for applications in which the sources are successively refinable.

In the broadcast strategy for the single user slowly fading Gaussian channel, the transmitter views the fading channel as a degraded Gaussian broadcast channel with a continuum of receivers. In [9] and [10] this broadcast problem is formulated for infinite fading levels and the throughput of the system is shown to be higher than the outage approach. On the other hand, Liu et. al. [4] show that 2-level superposition coding throughput approximates the infinite level superposition coding closely enough.

In this work, we apply the broadcast strategy to the slowly fading relay channel. Similar to [3] our relay receives and transmits on different channels. Based on the results of [4] we assume a 2-level superposition coding scheme in our system. Hence the transmitter follows a broadcast code, which is composed of superimposed information at two different rates designed for two fading levels. Thus the relay can decode partial information according to its channel quality to the transmitter. The relay then forwards that part of the information to the destination. The destination recovers the two levels of superimposed information combining the direct copy from the source and the signal from the relay. We show that utilizing the broadcast strategy increases the throughput of the relay channel with respect to amplify and forward or a modified version of decode and forward. The increase in the throughput is more emphasized if the relay has less power than the source. We also demonstrate that superposition of information, when coupled with a successively refinable source, decreases the overall expected distortion.

In the next section, we describe the system model and

<sup>1</sup>This material is based upon work partially supported by the National Science Foundation under Grant No. 0093163.

explain the schemes we compare in detail. In Sections III-A and III-B the throughput results are provided and the diversity levels for the broadcast approach are proved. Section III-C presents the expected distortion comparison of the schemes studied. We conclude in Section IV.

## II. SYSTEM MODEL

In our system, there is only one source-destination pair and one relay. We assume the source and the relay have orthogonal channels such as time division, shown in in Figure 1, taking into account that the relay cannot transmit and receive simultaneously. Here, the source transmits in the first time slot and both the destination and the relay listen to the source. Then in the next time slot the relay transmits and the destination listens to the relay. The source and the relay have power levels  $P_s$  and  $P_r$  respectively. We assume independent and identically distributed Rayleigh fading channels between every pair of nodes.

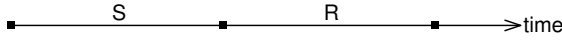


Fig. 1. Time division between the source and the relay

The source encodes its information using 2-level superposition coding. Therefore, the source signal can be written as the sum of two signals  $X_0$  and  $X_1$ :

$$X = X_0 + X_1. \quad (1)$$

Here  $X_0$  is the basic and  $X_1$  is the additional information. This superposition of information enables both the relay and the destination to decode the information successively at two different rates,  $R_0$  and  $R_1$ , whenever the equivalent channel condition is at least as good as the fading levels,  $|h_0|$  and  $|h_1|$  respectively. Here  $|h_0|$  denotes the bad channel state and  $|h_1|$  denotes the good channel state, where  $|h_0| \leq |h_1|$ . Then

$$R_0 = \ln \left( 1 + \frac{|h_0|^2 \bar{\alpha} P_s}{|h_0|^2 \alpha P_s + \mathcal{N}_o} \right) \quad (2)$$

$$R_1 = \ln \left( 1 + \frac{|h_1|^2 \alpha P_s}{\mathcal{N}_o} \right). \quad (3)$$

To decode  $X_0$ ,  $X_1$  is treated as noise. After decoding  $X_0$ , it is subtracted from the received signal and  $X_1$  can be decoded afterwards. If only  $X_0$  is decoded reliably, but  $X_1$  is in error, then the received rate is  $R_0$ . On the other hand, after decoding  $X_0$  correctly, if in addition  $X_1$  is decoded reliably, then the total received rate becomes  $R_0 + R_1$ .

We assume the total block length is  $N$ . In the first time slot, or first half of the block consisting of  $N/2$  transmissions, the source transmits the signal  $X$  in equation (1). The source allocates  $\alpha P_s$  power to  $X_1$  and  $\bar{\alpha} P_s$  power to  $X_0$ , where  $\bar{\alpha} = 1 - \alpha$  and  $\alpha \in [0, 1]$ . The corresponding received signals by the relay and the destination are  $Y_r$  and  $Y_d$  respectively. We have

$$Y_r = h_{sr}X + Z_r \quad (4)$$

$$Y_d = h_{sd}X + Z_d. \quad (5)$$

After receiving  $Y_r$  in the first time slot, the relay sends  $X_r$  resulting in the received signal  $Y'_d$  at the destination. If the relay observes a channel fading with amplitude less than  $|h_0|$ , then it cannot understand anything and hence cannot transmit any information during its time slot. If the source to relay channel fading is in between  $|h_0|$  and  $|h_1|$ , then the relay can reliably decode  $X_0$  and can allocate all of its power to  $X_0$ . If the relay understands everything, then it transmits both parts of the information  $X_0$  and  $X_1$  assigning power levels  $\bar{\alpha} P_r$  and  $\alpha P_r$  respectively.

$$X_r = \begin{cases} 0, & |h_{sr}| < |h_0| \\ \sqrt{\frac{P_r}{\bar{\alpha} P_s}} X_0, & |h_0| \leq |h_{sr}| < |h_1| \\ \sqrt{\frac{P_r}{P_s}} (X_0 + X_1), & |h_1| \leq |h_{sr}| \end{cases} \quad (6)$$

$$Y'_d = h_{rd}X_r + Z'_d \quad (7)$$

The coefficients  $h_{sd}$ ,  $h_{sr}$  and  $h_{rd}$  capture the effect of fading, which are assumed to be independent and identically distributed zero mean, complex Gaussian random variables with variance 1.  $Z_r$ ,  $Z_d$  and  $Z'_d$  denote complex Gaussian noise at the relay and at the destination respectively with variance  $\mathcal{N}_o$ .

Destination attempts to decode  $X_0$  first, combining  $X$  and  $X_r$ , which depends on the source to relay channel quality as in equation (6), and treating  $X_1$  as noise. We assume the destination knows which signal the relay has transmitted in equation (6). If the destination can successfully decode  $X_0$  then it attempts to decode  $X_1$  by subtracting off  $X_0$  first. This ordering is due to the layering of information in broadcast codes. If the destination cannot decode  $X_0$ , then it cannot decode  $X_1$  either.

We will compare this broadcast scheme with AF and a modified version of the DF in [3] which we call MDF. Different from the broadcast strategy in AF and MDF the source encodes the information at a fixed rate. In AF protocol the relay normalizes the power of  $Y_r$  to its own power constraint and simply forwards this normalized signal to the destination in the second time slot [3]. In MDF protocol, the relay tries to decode  $X$ . If the decoding is reliable the relay sends re-encoded information to the destination. Destination combines both the source and the relay signal. Even if relay cannot decode, destination uses the source signal. This is slightly different than the DF protocol presented in [3], in which the destination ignores the source signal when the relay cannot decode.

To have a fair comparison between the schemes described above, we use the expected rate criterion [10], since the broadcast strategy results in two different rates rather than a fixed rate. The outage probability of the broadcast approach for rate  $R_1$  is also an interesting measure. We look at rate  $R_1$  because whenever this additional information is not received reliably, it means the total information intended for the receiver is not received completely, regardless of whether  $R_0$  is received or not. In Section III we find the optimum expected rate for each scheme to study the throughput of the system. We also investigate the outage probability for rate  $R_1$ . In the same

section diversity orders of both information streams  $X_0$  and  $X_1$  are also discussed.

We also study the expected distortion of the broadcast strategy to see the overall performance over the communication system when a successively refinable source is present. We assume a Gaussian source with zero mean and unit variance and find the expected distortion as compared to AF and MDF in Section III-C.

### III. RESULTS

#### A. Throughput

For a simpler notation  $|h_{sd}|^2$ ,  $|h_{rd}|^2$  and  $|h_{sr}|^2$  are denoted by  $x$ ,  $y$  and  $z$  respectively. We define  $C(a)$ ,  $C_0(a)$  and  $C_1(a)$  to be

$$C(a) \triangleq \ln \left( 1 + a \frac{P_s}{\mathcal{N}_o} \right) \quad (8)$$

$$C_0(a) \triangleq \ln \left( 1 + \frac{a\bar{\alpha}P_s}{a\alpha P_s + \mathcal{N}_o} \right) \quad (9)$$

$$C_1(a) \triangleq \ln \left( 1 + a\alpha \frac{P_s}{\mathcal{N}_o} \right). \quad (10)$$

Now, the expected rate expression for the broadcast approach becomes:

$$ER_{BC} = P(\text{cannot decode } X_0)0 + P(\text{decode } X_0 \text{ only})R_0 + P(\text{decode } X_0 \text{ and } X_1)(R_0 + R_1). \quad (11)$$

This equation can also be written as

$$ER_{BC} = P_{out}(R_0)0 + [1 - P_{out}(R_0)]R_0 + [1 - P_{out}(R_1)]R_1. \quad (12)$$

Here

$$P_{out}(R_0) = P(C_0(z) < R_0)P(C_0(x) < R_0) + P(C_0(z) > R_0, C_1(z) < R_1) \times P \left( \ln \left( 1 + f \left( x \frac{P_s}{\mathcal{N}_o}, y \frac{P_r}{\mathcal{N}_o}, \alpha \right) \right) < R_0 \right) + P(C_1(z) > R_1)P(C_0(x + y \frac{P_r}{P_s}) < R_0) \quad (13)$$

is the probability of outage for  $R_0$  where

$$f \left( x \frac{P_s}{\mathcal{N}_o}, y \frac{P_r}{\mathcal{N}_o}, \alpha \right) = \frac{x\bar{\alpha} \frac{P_r}{\mathcal{N}_o}}{x\alpha \frac{P_s}{\mathcal{N}_o} + 1} + y \frac{P_r}{\mathcal{N}_o}$$

and

$$P_{out}(R_1) = P(C_1(z) < R_1)P(C_1(x) < R_1) + P(C_1(z) > R_1)P \left( C_1 \left( x + y \frac{P_r}{P_s} \right) < R_1 \right) \quad (14)$$

is the probability of outage for  $R_1$ . Note that if we have outage for additional rate  $R_1$ ,  $X_0$  may or may not be in outage. However, if  $X_0$  is in outage, the incremental information at rate  $R_1$ , which can only be decoded after subtracting  $X_0$  from the received signals, is always in outage.

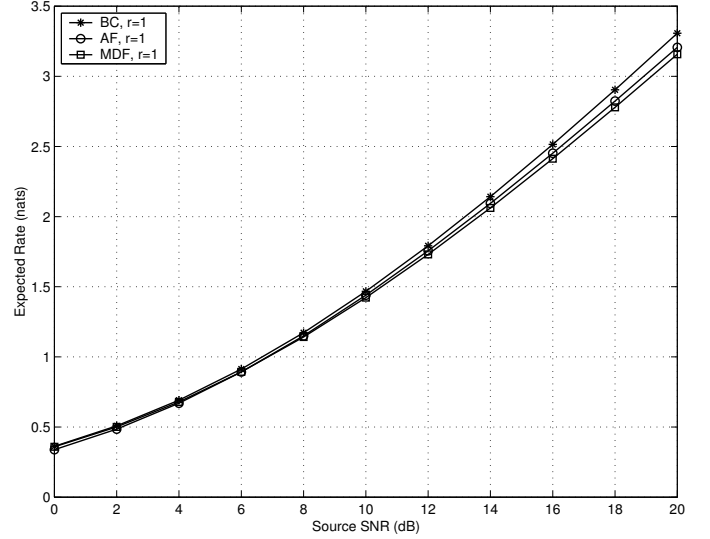


Fig. 2. Comparison of optimum BC, MDF and AF for  $r = 1$  ( $P_s = P_r$ )

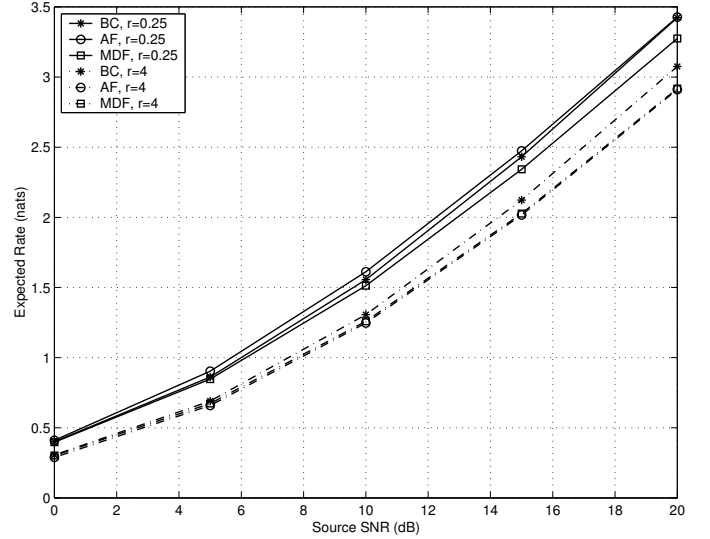


Fig. 3. Comparison of BC, MDF, AF for  $r = 0.25$  and  $r = 4$ . Here  $r$  denotes  $P_s/P_r$

For given  $P_s$  and  $P_r$  the expected rate expression in equation (11) depends on the good and bad channel states ( $|h_1|$  and  $|h_0|$ ) and power allocation  $\alpha$ . In order to find the optimum throughput for each SNR, we will maximize  $ER_{BC}$  over all these three parameters.

Similarly the expected rate expression for the MDF protocol is given by

$$ER_{MDF} = [1 - P_{out}(R)]R \quad (15)$$

where

$$P_{out}(R) = P(C(z) < R)P(C(x) < R) + P(C(z) > R)P \left( C \left( x + y \frac{P_r}{P_s} \right) < R \right) \quad (16)$$

and  $R$  is the fixed transmission rate. The expected rate expression for AF is same as equation (15); however,  $P_{out}(R)$

is different for AF, which is found in [3]. Now the optimization is over all rates  $R$  that maximize the expected rate.

We define source to relay power ratio as  $r = P_s/P_r$ . For  $r = 1$ ,  $P_s = P_r = P$ , we let  $\text{SNR} = P/\mathcal{N}_o$  and illustrate our results for the expected rate as a function of SNR in Figure 2. Although finding the optimum  $R$  values for AF and MDF are analytically intractable, we can show that the optimum threshold  $s$ , where  $s = (e^R - 1)/\text{SNR}$ , is less than or equal to 1. This observation, which simplifies our numerical search, is proved in the Appendix. Based on our results for AF and MDF, we conjecture that the optimum  $|h_0|$  and  $|h_1|$  values for the broadcast strategy are also less than 1. The optimum  $|h_0|$  and  $|h_1|$  values in the interval  $[0, 1]$  along with the best power allocation  $\alpha$  for various SNR values are shown in Table I.

TABLE I  
THE  $|h_0|$ ,  $|h_1|$  AND  $\alpha$  VALUES FOR EACH SNR

SNR (dB)	$ h_0 $	$ h_1 $	$\alpha$
0	0.86	0.97	0.34
2	0.82	0.95	0.32
4	0.79	0.94	0.27
6	0.75	0.92	0.24
8	0.72	0.90	0.20
10	0.68	0.88	0.17
12	0.65	0.86	0.14
14	0.61	0.84	0.11
16	0.58	0.82	0.09
18	0.56	0.81	0.07
20	0.53	0.80	0.05

One could expect to find increasing  $\alpha$  values and the broadcast strategy to approach the MDF protocol for increasing SNR. However, it is interesting to observe that with increasing SNR,  $\alpha$  decreases, whereas  $\alpha\text{SNR}$  product increases.

From Figure 2 it can be observed that the broadcast strategy results in the highest throughput over the relay channel. The difference between the broadcast strategy and the other protocols increase with increasing SNR. The total rate of the broadcast strategy  $R_0 + R_1$  evaluated at  $\alpha \neq 1$  is always less than the total rate at  $\alpha = 1$ . Despite this rate loss, the broadcast strategy allows partial information to be decoded reliably at the destination more often.

For  $r = 0.25$  and  $r = 4$ , based on our results for  $r = 1$  we choose the threshold values  $|h_0|$ ,  $|h_1|$  and  $s$  in the interval  $[0, 1]$ . The results are shown in Figure 3. We observe that the gains of the broadcast strategy are emphasized more if the relay power is less than the source power. In a general wireless network, if the relay allocates less power to relaying or in time as the relay power depletes, using the broadcast strategy is more advantageous.

### B. Diversity

In this subsection we prove that both of the information streams in the broadcast strategy have 2-levels of diversity for fixed  $R_0$ ,  $R_1$  and  $\alpha$ . Hence broadcast approach provides higher throughput than AF and MDF without loss in diversity. For demonstration purposes we give the proof for equal source

and relay powers but the results easily generalize to different  $r$  values. To prove 2-levels of diversity for the broadcast strategy, we show that the outage expressions for both streams  $P_{out}(R_0)$  and  $P_{out}(R_1)$  are inversely proportional to  $\text{SNR}^2$  as SNR goes to infinity. We define  $b(\text{SNR})$  and  $g(\text{SNR})$  to be

$$b(\text{SNR}) = \frac{(e^{R_0} - 1)}{(1 - e^{R_0}\alpha)\text{SNR}} \quad (17)$$

$$g(\text{SNR}) = \frac{e^{R_1} - 1}{\alpha\text{SNR}}. \quad (18)$$

To find the diversity order of  $R_0$ , we need to find the limiting behavior of the terms in equation (13). Using the results of [3], we can find that

$$\lim_{\text{SNR} \rightarrow \infty} \frac{1}{b(\text{SNR})} \text{P}(C_0(x) < R_0) = 1 \quad (19)$$

$$\lim_{\text{SNR} \rightarrow \infty} \frac{1}{b^2(\text{SNR})} \text{P}(C_0(x+y) < R_0) = \frac{1}{2}$$

$$\lim_{\text{SNR} \rightarrow \infty} \text{P}(C_1(x) > R_1) = 1$$

and  $\text{P}(C_0(z) < R_0)$  behaves like equation (19). On the other hand

$$\text{P}(C_0(z) > R_0, C_1(z) < R_1) \leq \text{P}(C_1(z) < R_1)$$

and

$$\lim_{\text{SNR} \rightarrow \infty} \frac{1}{g(\text{SNR})} \text{P}(C_1(z) < R_1) = 1.$$

Similarly

$$\text{P}\left(\ln\left(1 + f\left(x\frac{P}{\mathcal{N}_o}, y\frac{P}{\mathcal{N}_o}, \alpha\right)\right) < R_0\right) \leq \text{P}(C_0(x+y) < R_0).$$

Combining for the probability of outage term  $P_{out}(R_0)$  we have

$$\lim_{\text{SNR} \rightarrow \infty} \frac{1}{b^2(\text{SNR})} P_{out}(R_0) \leq \frac{3}{2}.$$

This implies that this scheme provides at least two levels of diversity for  $R_0$ . Since this scheme cannot provide more than two levels of diversity we conclude that  $R_0$  stream has two levels of diversity.

For  $R_1$  the diversity analysis is due to [3] and

$$\lim_{\text{SNR} \rightarrow \infty} \frac{1}{g^2(\text{SNR})} P_{out}(R_1) = \frac{3}{2}.$$

This means  $R_1$  stream also has two levels of diversity. Hence we conclude that both information streams have 2-levels of diversity.

### C. Distortion

As discussed before, the broadcast strategy is very suitable to transmit successively refinable sources. In this subsection we assume a Gaussian source with zero mean and unit variance and study the overall source distortion when the broadcast strategy is employed and the source and the relay have

equal power levels. The expected distortion expression can be written as:

$$\begin{aligned}
 ED_{BC} &= P(\text{cannot decode } X_0)1 \\
 &+ P(\text{decode } X_0 \text{ only})e^{-R_0} + \\
 &+ P(\text{decode } X_0 \text{ and } X_1)e^{-R_0-R_1}. \quad (20)
 \end{aligned}$$

Similar expressions for AF and MDF can be written. In Figure 4 we calculate the minimum expected distortion over all  $\alpha$ ,  $R_0$ ,  $R_1$  and  $R$ . We compare the results of the broadcast strategy with the optimum expected distortion of MDF, AF, direct transmission and direct transmission with broadcast. For direct transmission the relay is not employed and the source transmits fresh information on its own at each time slot. For direct with broadcast, this fresh information, this fresh signal consists of two levels of superimposed information. From Figure 4 we observe that the broadcast approach performs much better than MDF and AF. Coupling a successively refinable source with the broadcast strategy emphasizes the importance of receiving partial information more. By comparing the broadcast strategy to the direct transmission, we observe that it gives us an advantageous relaying scheme that introduces gains over direct transmission, whereas the other relaying schemes AF and MDF perform worse than direct as observed in [2]. This is because in AF and MDF protocols diversity gains cannot compensate for spectral efficiency losses in a time-division scheme. On the other hand, the broadcast strategy performs worse than direct transmission with broadcast, which is expected given the results of [2].

In [2] the decay rate of expected distortion with respect to SNR as SNR approaches infinity is also studied. An interesting future direction would be to investigate this for the broadcast strategy and for direct transmission with broadcast.

#### IV. CONCLUSION

In this work we apply the broadcast strategy of fading channels to a communication system that has a relay. With this strategy, the source information is encoded into two streams as a 2-level broadcast code corresponding to the “bad” and “good” channel states. Superposition of information allows both the relay and the destination to partially decode the source information. This way the relay noise propagation is eliminated as opposed to AF and the relay is involved in the system more often than it is in MDF. We compare our results with AF protocol and MDF and show that using superposition of information results in higher expected rates. This improvement is larger if the source to relay power ratio is less than 1. In addition to this, we show that both streams of the broadcast strategy have two levels of diversity. Therefore, the broadcast method has the potential to improve throughput while attaining maximum diversity of cooperative relaying. The broadcast approach, when coupled with a successively refinable source, also decreases the expected distortion of the system compared to AF and MDF.

Applying the broadcast strategy to relay channels still has open problems such as finding the optimum  $|h_0|$ ,  $|h_1|$ , and  $\alpha$

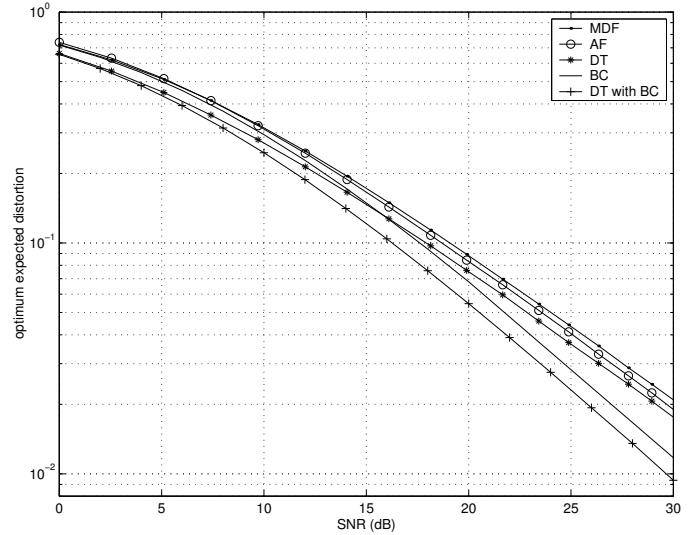


Fig. 4. Optimum expected distortion versus SNR for MDF, AF, direct transmission, broadcast over relay, direct transmission with broadcast

values as a function of SNR and more generally the diversity-multiplexing trade-off for variable  $|h_0|$ ,  $|h_1|$ , and  $\alpha$ .

#### V. APPENDIX

In this appendix, we show that for MDF and AF and for  $P_s = P_r = P$ , the optimum threshold  $s$ , where  $s = (e^R - 1)/\text{SNR}$ , is always less than or equal to 1. Using equations (15) and (16) the expected rate expression for the MDF protocol can be simplified into

$$ER_{MDF} = e^{-s} (1 + se^{-s}) \ln(1 + s\text{SNR}).$$

To show the optimum threshold  $s$  that maximizes the  $ER_{MDF}$  expression is less than 1, we consider  $\frac{d}{ds}ER_{MDF}$ .

$$\begin{aligned}
 \frac{d}{ds}ER_{MDF} &= -e^{-s} (1 + se^{-s}) \ln(1 + s\text{SNR}) \\
 &+ e^{-s} (e^{-s} - se^{-s}) \ln(1 + s\text{SNR}) \\
 &+ e^{-s} (1 + se^{-s}) \frac{\text{SNR}}{1 + s\text{SNR}}
 \end{aligned}$$

When  $s$  is greater than 1, the second term in the sum is negative and the magnitude of the first term is larger than the magnitude of the last term in the above expression. Hence the derivative of  $ER_{MDF}$  is negative if  $s$  is greater than 1 and the optimum  $s$  must be less than or equal to 1.

Using the same notation as Section III-A, the probability of outage expression for the AF protocol is:

$$P_{out}(R) = P(\ln(1 + x\text{SNR} + h(y\text{SNR}, z\text{SNR})) < R)$$

where

$$h(x, y) = \frac{xy}{x + y + 1}.$$

Then the expected rate expression as a function of  $s$  is

$$\begin{aligned}
 ER_{AF}(s) &= P\left(x + \frac{1}{\text{SNR}}h(y\text{SNR}, z\text{SNR}) > s\right) \\
 &\times \ln(1 + s\text{SNR}).
 \end{aligned}$$

Since the function  $h(x, y)$  is always less than or equal to  $\min\{x, y\}$ , we have the following upper bound on  $ER_{AF}(s)$ :

$$\begin{aligned} ER_{AF}(s) &\leq ER_U(s) \\ &= e^{-s}(1+s)\ln(1+s\text{SNR}). \end{aligned}$$

the upper bound is a decreasing function of  $s$ , if  $s$  is large enough. To find an upper bound on the optimum threshold  $s$  for the AF protocol we follow a numeric approach. We find two threshold values  $s_0$  and  $s_1$  such that the upper bound  $ER_U(s)$  is a decreasing function for all  $s$  greater than or equal to  $s_1$  and  $ER_{AF}(s_0)$  is greater than  $ER_U(s_1)$ . This implies the optimum threshold for the AF protocol is less than  $s_1$ . We have searched for all  $s$  values in the interval  $[0, s_1]$  and still observed that the optimum threshold is less than 1, although in general  $s_1$  can be larger than 1. Thus we conclude both MDF and AF protocols have optimum threshold values  $s$  less than or equal to 1.

#### REFERENCES

- [1] T. M. Cover. Broadcast channels. *IEEE Transactions on Information Theory*, 18(1):2, January 1972.
- [2] D. Gunduz and E. Erkip. Joint source-channel cooperation: Diversity versus spectral efficiency. In *Proceedings of IEEE International Symposium on Information Theory*, June 27- July 2, 2004.
- [3] J.N. Laneman, D. N. C. Tse, and G. W. Wornell. Cooperative diversity in wireless networks: Efficient protocols and outage behavior. *IEEE Transactions on Information Theory*. To appear.
- [4] Y. Liu, K. N. Lau, O. Y. Takeshita, and M. P. Fitz. Optimal rate allocation for superposition coding in quasi-static fading channels. In *Proceedings of IEEE International Symposium on Information Theory*, June 30- July 5, 2002.
- [5] L. H. Ozarow, S. Shamai (Shitz), and A. D. Wyner. Information theoretic considerations for cellular mobile radio. *IEEE Transactions on Vehicular Technology*, 43:359, May 1994.
- [6] J. G. Proakis. *Digital Communications*. McGraw-Hill, Inc., New York, Fourth edition, 2000.
- [7] A. Sendonaris, E. Erkip, and B. Aazhang. User cooperation diversity-Part I: System description. *IEEE Transactions on Communications*, 51(11):1927, November 2003.
- [8] A. Sendonaris, E. Erkip, and B. Aazhang. User cooperation diversity-Part II: Implementation aspects and performance analysis. *IEEE Transactions on Communications*, 51(11):1939, November 2003.
- [9] S. Shamai (Shitz). A broadcast strategy for the Gaussian slowly fading channel. In *Proceedings of IEEE International Symposium on Information Theory*, June 29- July 4, 1997.
- [10] S. Shamai (Shitz) and A. Steiner. A broadcast approach for a single-user slowly fading MIMO channel. *IEEE Transactions on Information Theory*, 49(10):2617, October 2003.